3

CHAPTER

PROBABILITY

The television game show *The Price Is Right* presents a wide range of pricing games in which contestants compete for prizes using strategy, probability, and their knowledge of prices. One popular game is *Spelling Bee*.

WHERE YOU'VE BEEN

In Chapters 1 and 2, you learned how to collect and describe data. Once the data are collected and described, you can use the results to write summaries, form conclusions, and make decisions. For instance, in *Spelling Bee*, contestants have a chance to win a car by choosing lettered cards that spell CAR or by choosing a single card that displays the entire word CAR. By collecting and analyzing data, you can determine the chances of winning the car.

To play *Spelling Bee*, contestants choose from 30 cards. Eleven cards display the letter C, eleven cards display A, six cards display R, and two cards display CAR. Depending on how well contestants play the game, they can choose two, three, four, or five cards.

Before the chosen cards are displayed, contestants are offered \$1000 for each card. If contestants choose the money, the game is over. If contestants choose to try to win the car, the host displays one card. After a card is displayed, contestants are offered \$1000 for each remaining card. If they do not accept the money, the host continues displaying cards. Play continues until contestants take the money, spell the word CAR, display the word CAR, or display all cards and do not spell CAR.

WHERE YOU'RE GOING

In Chapter 3, you will learn how to determine the probability of an event. For instance, the following table shows the four ways that contestants on *Spelling Bee* can win a car and the corresponding probabilities.

You can see from the table that choosing more cards gives you a better chance of winning. These probabilities can be found using *combinations*, which will be discussed in Section 3.4.

Event	Probability
Winning by selecting two cards	$\frac{57}{435} \approx 0.131$
Winning by selecting three cards	$\frac{151}{406} \approx 0.372$
Winning by selecting four cards	$\frac{1067}{1827} \approx 0.584$
Winning by selecting five cards	$\frac{52{,}363}{71{,}253} \approx 0.735$

3.1 Basic Concepts of Probability and Counting

WHAT YOU SHOULD LEARN

- How to identify the sample space of a probability experiment and how to identify simple events
- How to use the Fundamental Counting Principle to find the number of ways two or more events can occur
- How to distinguish among classical probability, empirical probability, and subjective probability
- How to find the probability of the complement of an event
- How to use a tree diagram and the Fundamental Counting Principle to find more probabilities

Probability Experiments ▸ The Fundamental Counting Principle ▸ Types of Probability ▸ Complementary Events ▸ Probability Applications

PROBABILITY EXPERIMENTS

When weather forecasters say that there is a 90% chance of rain or a physician says there is a 35% chance for a successful surgery, they are stating the likelihood, or *probability*, that a specific event will occur. Decisions such as "should you go golfing" or "should you proceed with surgery" are often based on these probabilities. In the previous chapter, you learned about the role of the descriptive branch of statistics. Because probability is the foundation of inferential statistics, it is necessary to learn about probability before proceeding to the second branch—inferential statistics.

DEFINITION

A **probability experiment** is an action, or trial, through which specific results (counts, measurements, or responses) are obtained. The result of a single trial in a probability experiment is an **outcome.** The set of all possible outcomes of a probability experiment is the **sample space.** An **event** is a subset of the sample space. It may consist of one or more outcomes.

STUDY TIP

Here is a simple example of the use of the terms *probability experiment*, *sample space*, *event*, and *outcome*.

Probability Experiment:
Roll a six-sided die.

Sample Space:
{1, 2, 3, 4, 5, 6}

Event:
Roll an even number, {2, 4, 6}.

Outcome:
Roll a 2, {2}.

EXAMPLE 1

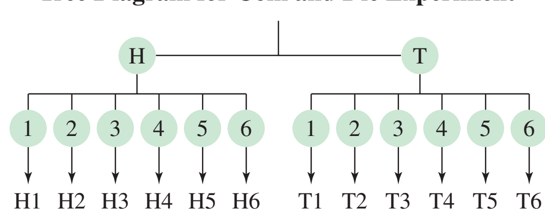
▸ Identifying the Sample Space of a Probability Experiment

A probability experiment consists of tossing a coin and then rolling a six-sided die. Determine the number of outcomes and identify the sample space.

▸ Solution

There are two possible outcomes when tossing a coin: a head (H) or a tail (T). For each of these, there are six possible outcomes when rolling a die: 1, 2, 3, 4, 5, or 6. A **tree diagram** gives a visual display of the outcomes of a probability experiment by using branches that originate from a starting point. It can be used to find the number of possible outcomes in a sample space as well as individual outcomes.

Tree Diagram for Coin and Die Experiment



From the tree diagram, you can see that the sample space has 12 outcomes.

{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

▸ Try It Yourself 1

For each probability experiment, determine the number of outcomes and identify the sample space.

1. A probability experiment consists of recording a response to the survey statement at the left *and* the gender of the respondent.
2. A probability experiment consists of recording a response to the survey statement at the left *and* the geographic location (Northeast, South, Midwest, West) of the respondent.

a. Start a *tree diagram* by forming a branch for each possible response to the survey.
b. At the end of each survey response branch, draw a *new branch* for each possible outcome.
c. Find the *number of outcomes* in the sample space.
d. List the *sample space*.

Answer: Page A34

In the rest of this chapter, you will learn how to calculate the probability or likelihood of an event. Events are often represented by uppercase letters, such as A, B, and C. An event that consists of a single outcome is called a **simple event.** In Example 1, the event "tossing heads and rolling a 3" is a simple event and can be represented as $A = \{H3\}$. In contrast, the event "tossing heads and rolling an even number" is not simple because it consists of three possible outcomes $B = \{H2, H4, H6\}$.

EXAMPLE 2

▸ Identifying Simple Events

Determine the number of outcomes in each event. Then decide whether each event is simple or not. Explain your reasoning.

1. For quality control, you randomly select a machine part from a batch that has been manufactured that day. Event A is selecting a specific defective machine part.

2. You roll a six-sided die. Event B is rolling at least a 4.

▸ Solution

1. Event A has only one outcome: choosing the specific defective machine part. So, the event is a simple event.

2. Event B has three outcomes: rolling a 4, a 5, or a 6. Because the event has more than one outcome, it is not simple.

▸ Try It Yourself 2

You ask for a student's age at his or her last birthday. Determine the number of outcomes in each event. Then decide whether each event is simple or not. Explain your reasoning.

1. Event C: The student's age is between 18 and 23, inclusive.
2. Event D: The student's age is 20.

a. Determine the number of *outcomes* in the event.
b. State whether the event is *simple* or not. Explain your reasoning.

Answer: Page A34

THE FUNDAMENTAL COUNTING PRINCIPLE

In some cases, an event can occur in so many different ways that it is not practical to write out all the outcomes. When this occurs, you can rely on the Fundamental Counting Principle. The Fundamental Counting Principle can be used to find the number of ways two or more events can occur in sequence.

THE FUNDAMENTAL COUNTING PRINCIPLE

If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$. This rule can be extended to any number of events occurring in sequence.

In words, the number of ways that events can occur in sequence is found by multiplying the number of ways one event can occur by the number of ways the other event(s) can occur.

EXAMPLE 3

Using the Fundamental Counting Principle

You are purchasing a new car. The possible manufacturers, car sizes, and colors are listed.

Manufacturer:	Ford, GM, Honda
Car size:	compact, midsize
Color:	white (W), red (R), black (B), green (G)

How many different ways can you select one manufacturer, one car size, and one color? Use a tree diagram to check your result.

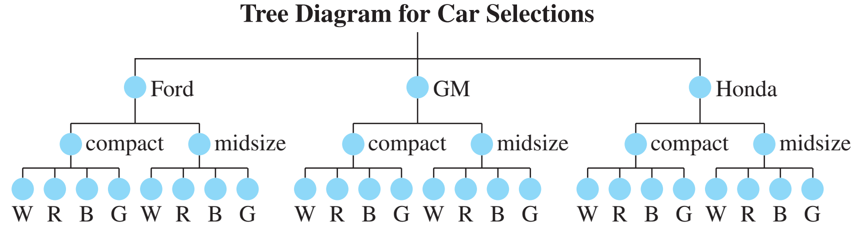
Solution

There are three choices of manufacturers, two choices of car sizes, and four choices of colors. Using the Fundamental Counting Principle, you can conclude that the number of ways to select one manufacturer, one car size, and one color is

$$3 \cdot 2 \cdot 4 = 24 \text{ ways}.$$

Using a tree diagram, you can see why there are 24 options.

Tree Diagram for Car Selections



Try It Yourself 3

Your choices now include a Toyota and a tan car. How many different ways can you select one manufacturer, one car size, and one color? Use a tree diagram to check your result.

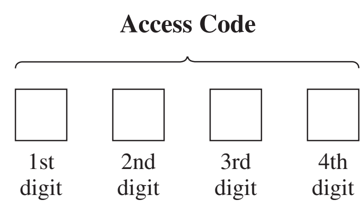
a. Find the *number of ways* each event can occur.
b. Use the *Fundamental Counting Principle*.
c. Use a *tree diagram* to check your result.

Answer: Page A35

EXAMPLE 4

Using the Fundamental Counting Principle

The access code for a car's security system consists of four digits. Each digit can be any number from 0 through 9.



How many access codes are possible if

1. each digit can be used only once and not repeated?

2. each digit can be repeated?

3. each digit can be repeated but the first digit cannot be 0 or 1?

Solution

1. Because each digit can be used only once, there are 10 choices for the first digit, 9 choices left for the second digit, 8 choices left for the third digit, and 7 choices left for the fourth digit. Using the Fundamental Counting Principle, you can conclude that there are

$$10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

possible access codes.

2. Because each digit can be repeated, there are 10 choices for each of the four digits. So, there are

$$\begin{aligned} 10 \cdot 10 \cdot 10 \cdot 10 &= 10^4 \\ &= 10{,}000 \end{aligned}$$

possible access codes.

3. Because the first digit cannot be 0 or 1, there are 8 choices for the first digit. Then there are 10 choices for each of the other three digits. So, there are

$$8 \cdot 10 \cdot 10 \cdot 10 = 8000$$

possible access codes.

Try It Yourself 4

How many license plates can you make if a license plate consists of

1. six (out of 26) alphabetical letters each of which can be repeated?
2. six (out of 26) alphabetical letters each of which cannot be repeated?
3. six (out of 26) alphabetical letters each of which can be repeated but the first letter cannot be A, B, C, or D?

a. *Identify* each event and the *number of ways* each event can occur.
b. Use the *Fundamental Counting Principle*. *Answer: Page A35*

TYPES OF PROBABILITY

The method you will use to calculate a probability depends on the type of probability. There are three types of probability: *classical probability*, *empirical probability*, and *subjective probability*. The probability that event E will occur is written as $P(E)$ and is read "the probability of event E."

STUDY TIP

Probabilities can be written as fractions, decimals, or percents. In Example 5, the probabilities are written as fractions and decimals, rounded when necessary to three places. This *round-off rule* will be used throughout the text.

DEFINITION

Classical (or **theoretical**) **probability** is used when each outcome in a sample space is equally likely to occur. The classical probability for an event E is given by

$$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}.$$

EXAMPLE 5

Finding Classical Probabilities

You roll a six-sided die. Find the probability of each event.

1. Event A: rolling a 3
2. Event B: rolling a 7
3. Event C: rolling a number less than 5

Solution

When a six-sided die is rolled, the sample space consists of six outcomes: $\{1, 2, 3, 4, 5, 6\}$.

1. There is one outcome in event $A = \{3\}$. So,

$$P(\text{rolling a } 3) = \frac{1}{6} \approx 0.167.$$

2. Because 7 is not in the sample space, there are no outcomes in event B. So,

$$P(\text{rolling a } 7) = \frac{0}{6} = 0.$$

3. There are four outcomes in event $C = \{1, 2, 3, 4\}$. So,

$$P(\text{rolling a number less than } 5) = \frac{4}{6} = \frac{2}{3} \approx 0.667.$$

Try It Yourself 5

You select a card from a standard deck. Find the probability of each event.

1. Event D: Selecting a nine of clubs
2. Event E: Selecting a heart
3. Event F: Selecting a diamond, heart, club, or spade

a. Identify the *total number of outcomes* in the sample space.
b. Find the *number of outcomes* in the event.
c. Use the *classical probability formula*.

Answer: Page A35

Standard Deck of Playing Cards

Hearts	Diamonds	Spades	Clubs
A ♥	A ♦	A ♠	A ♣
K ♥	K ♦	K ♠	K ♣
Q ♥	Q ♦	Q ♠	Q ♣
J ♥	J ♦	J ♠	J ♣
10 ♥	10 ♦	10 ♠	10 ♣
9 ♥	9 ♦	9 ♠	9 ♣
8 ♥	8 ♦	8 ♠	8 ♣
7 ♥	7 ♦	7 ♠	7 ♣
6 ♥	6 ♦	6 ♠	6 ♣
5 ♥	5 ♦	5 ♠	5 ♣
4 ♥	4 ♦	4 ♠	4 ♣
3 ♥	3 ♦	3 ♠	3 ♣
2 ♥	2 ♦	2 ♠	2 ♣

When an experiment is repeated many times, regular patterns are formed. These patterns make it possible to find empirical probability. Empirical probability can be used even if each outcome of an event is not equally likely to occur.

PICTURING THE WORLD

It seems as if no matter how strange an event is, somebody wants to know the probability that it will occur. The following table lists the probabilities that some intriguing events will happen. (Adapted from Life: The Odds)

Event	Probability
Being audited by the IRS	0.6%
Writing a *New York Times* best seller	0.0045
Winning an Academy Award	0.000087
Having your identity stolen	0.5%
Spotting a UFO	0.0000003

Which of these events is most likely to occur? Least likely?

To explore this topic further, see Activity 3.1 on page 144.

DEFINITION

Empirical (or **statistical**) **probability** is based on observations obtained from probability experiments. The empirical probability of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}}$$
$$= \frac{f}{n}$$

EXAMPLE 6

Finding Empirical Probabilities

A company is conducting a telephone survey of randomly selected individuals to get their overall impressions of the past decade (2000s). So far, 1504 people have been surveyed. The frequency distribution shows the results. What is the probability that the next person surveyed has a positive overall impression of the 2000s? *(Adapted from Princeton Survey Research Associates International)*

Response	Number of times, f
Positive	406
Negative	752
Neither	316
Don't know	30
	$\Sigma f = 1504$

Solution

The event is a response of "positive." The frequency of this event is 406. Because the total of the frequencies is 1504, the empirical probability of the next person having a positive overall impression of the 2000s is

$$P(\text{positive}) = \frac{406}{1504}$$
$$\approx 0.270.$$

Try It Yourself 6

An insurance company determines that in every 100 claims, 4 are fraudulent. What is the probability that the next claim the company processes will be fraudulent?

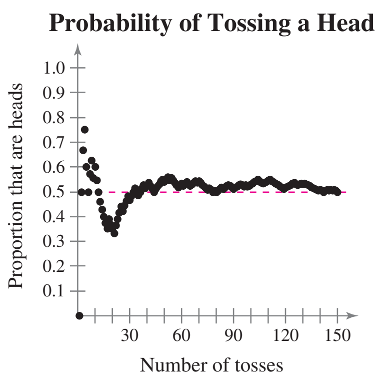
a. *Identify* the event. Find the *frequency* of the event.
b. Find the *total frequency* for the experiment.
c. Find the *empirical probability* of the event.

Answer: Page A35

As you increase the number of times a probability experiment is repeated, the empirical probability (relative frequency) of an event approaches the theoretical probability of the event. This is known as the **law of large numbers.**

LAW OF LARGE NUMBERS

As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.



As an example of this law, suppose you want to determine the probability of tossing a head with a fair coin. If you toss the coin 10 times and get only 3 heads, you obtain an empirical probability of $\frac{3}{10}$. Because you tossed the coin only a few times, your empirical probability is not representative of the theoretical probability, which is $\frac{1}{2}$. If, however, you toss the coin several thousand times, then the law of large numbers tells you that the empirical probability will be very close to the theoretical or actual probability.

The scatter plot at the left shows the results of simulating a coin toss 150 times. Notice that, as the number of tosses increases, the probability of tossing a head gets closer and closer to the theoretical probability of 0.5.

EXAMPLE 7

Using Frequency Distributions to Find Probabilities

Employee ages	Frequency, f
15 to 24	54
25 to 34	366
35 to 44	233
45 to 54	180
55 to 64	125
65 and over	42
	$\Sigma f = 1000$

You survey a sample of 1000 employees at a company and record the age of each. The results are shown in the frequency distribution at the left. If you randomly select another employee, what is the probability that the employee will be between 25 and 34 years old?

Solution

The event is selecting an employee who is between 25 and 34 years old. The frequency of this event is 366. Because the total of the frequencies is 1000, the empirical probability of selecting an employee between the ages of 25 and 34 years old is

$$P(\text{age 25 to 34}) = \frac{366}{1000} = 0.366.$$

Try It Yourself 7

Find the probability that an employee chosen at random will be between 15 and 24 years old.

a. Find the *frequency* of the event.
b. Find the *total of the frequencies*.
c. Find the *empirical probability* of the event.

Answer: Page A35

The third type of probability is **subjective probability.** Subjective probabilities result from intuition, educated guesses, and estimates. For instance, given a patient's health and extent of injuries, a doctor may feel that the patient has a 90% chance of a full recovery. Or a business analyst may predict that the chance of the employees of a certain company going on strike is 0.25.

EXAMPLE 8

Classifying Types of Probability

Classify each statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.

1. The probability that you will get the flu this year is 0.1.
2. The probability that a voter chosen at random will be younger than 35 years old is 0.3.
3. The probability of winning a 1000-ticket raffle with one ticket is $\frac{1}{1000}$.

Solution

1. This probability is most likely based on an educated guess. It is an example of subjective probability.
2. This statement is most likely based on a survey of a sample of voters, so it is an example of empirical probability.
3. Because you know the number of outcomes and each is equally likely, this is an example of classical probability.

Try It Yourself 8

Based on previous counts, the probability of a salmon successfully passing through a dam on the Columbia River is 0.85. Is this statement an example of classical probability, empirical probability, or subjective probability? *(Source: Army Corps of Engineers)*

a. Identify the *event*.
b. Decide whether the probability is *determined* by knowing all possible outcomes, whether the probability is *estimated* from the results of an experiment, or whether the probability is an *educated guess*.
c. Make a *conclusion*.

Answer: Page A35

A probability cannot be negative or greater than 1. So, the probability of an event E is between 0 and 1, inclusive, as stated in the following rule.

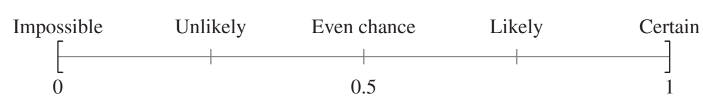
RANGE OF PROBABILITIES RULE

The probability of an event E is between 0 and 1, inclusive. That is,

$$0 \leq P(E) \leq 1.$$

If the probability of an event is 1, the event is certain to occur. If the probability of an event is 0, the event is impossible. A probability of 0.5 indicates that an event has an even chance of occurring.

The following graph shows the possible range of probabilities and their meanings.



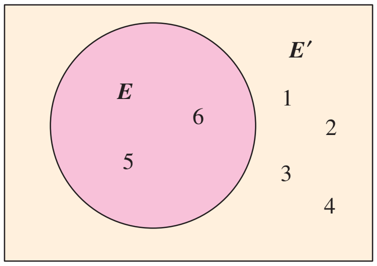
An event that occurs with a probability of 0.05 or less is typically considered unusual. Unusual events are highly unlikely to occur. Later in this course you will identify unusual events when studying inferential statistics.

COMPLEMENTARY EVENTS

The sum of the probabilities of all outcomes in a sample space is 1 or 100%. An important result of this fact is that if you know the probability of an event E, you can find the probability of the *complement of event E*.

DEFINITION

The **complement of event *E*** is the set of all outcomes in a sample space that are not included in event E. The complement of event E is denoted by E' and is read as "E prime."



The area of the rectangle represents the total probability of the sample space $(1 = 100\%)$. The area of the circle represents the probability of event E, and the area outside the circle represents the probability of the complement of event E.

For instance, if you roll a die and let E be the event "the number is at least 5," then the complement of E is the event "the number is less than 5." In symbols, $E = \{5, 6\}$ and $E' = \{1, 2, 3, 4\}$.

Using the definition of the complement of an event and the fact that the sum of the probabilities of all outcomes is 1, you can determine the following formulas.

$$P(E) + P(E') = 1 \qquad P(E) = 1 - P(E') \qquad P(E') = 1 - P(E)$$

The Venn diagram at the left illustrates the relationship between the sample space, an event E, and its complement E'.

EXAMPLE 9

Finding the Probability of the Complement of an Event

Use the frequency distribution in Example 7 to find the probability of randomly choosing an employee who is not between 25 and 34 years old.

Solution

From Example 7, you know that

$$P(\text{age 25 to 34}) = \frac{366}{1000} = 0.366.$$

So, the probability that an employee is not between 25 and 34 years old is

$$P(\text{age is not 25 to 34}) = 1 - \frac{366}{1000} = \frac{634}{1000} = 0.634.$$

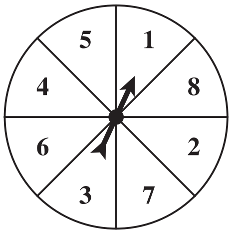
Try It Yourself 9

Use the frequency distribution in Example 7 to find the probability of randomly choosing an employee who is not between 45 and 54 years old.

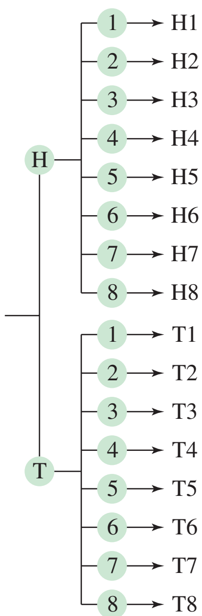
a. Find the *probability* of randomly choosing an employee who is between 45 and 54 years old.
b. *Subtract* the resulting probability from 1.
c. *State the probability* as a fraction and as a decimal.

Answer: Page A35

PROBABILITY APPLICATIONS



Tree Diagram for Coin and Spinner Experiment



EXAMPLE 10

Using a Tree Diagram

A probability experiment consists of tossing a coin and spinning the spinner shown at the left. The spinner is equally likely to land on each number. Use a tree diagram to find the probability of each event.

1. Event A: tossing a tail and spinning an odd number

2. Event B: tossing a head or spinning a number greater than 3

Solution From the tree diagram at the left, you can see that there are 16 outcomes.

1. There are four outcomes in event $A = \{T1, T3, T5, T7\}$. So,

$$P(\text{tossing a tail and spinning an odd number}) = \frac{4}{16} = \frac{1}{4} = 0.25.$$

2. There are 13 outcomes in event $B = \{H1, H2, H3, H4, H5, H6, H7, H8, T4, T5, T6, T7, T8\}$. So,

$$P(\text{tossing a head or spinning a number greater than 3}) = \frac{13}{16} \approx 0.813.$$

Try It Yourself 10

Find the probability of tossing a tail and spinning a number less than 6.

a. Find the *number of outcomes* in the event.
b. Find the *probability of the event*.

Answer: Page A35

EXAMPLE 11

Using the Fundamental Counting Principle

Your college identification number consists of eight digits. Each digit can be 0 through 9 and each digit can be repeated. What is the probability of getting your college identification number when randomly generating eight digits?

Solution Because each digit can be repeated, there are 10 choices for each of the 8 digits. So, using the Fundamental Counting Principle, there are $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^8 = 100{,}000{,}000$ possible identification numbers. But only one of those numbers corresponds to your college identification number. So, the probability of randomly generating 8 digits and getting your college identification number is 1/100,000,000.

Try It Yourself 11

Your college identification number consists of nine digits. The first two digits of each number will be the last two digits of the year you are scheduled to graduate. The other digits can be any number from 0 through 9, and each digit can be repeated. What is the probability of getting your college identification number when randomly generating the other seven digits?

a. Find the *total number* of possible identification numbers. Assume that you are scheduled to graduate in 2015.
b. Find the *probability* of randomly generating your identification number.

Answer: Page A35

3.1 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. What is the difference between an outcome and an event?

2. Determine which of the following numbers could not represent the probability of an event. Explain your reasoning.

(a) 33.3% (b) -1.5 (c) 0.0002 (d) 0 (e) $\frac{320}{1058}$ (f) $\frac{64}{25}$

3. Explain why the following statement is incorrect: *The probability of rain tomorrow is 150%*.

4. When you use the Fundamental Counting Principle, what are you counting?

5. Use your own words to describe the law of large numbers. Give an example.

6. List the three formulas that can be used to describe complementary events.

True or False? *In Exercises 7–10, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

7. If you roll a six-sided die six times, you will roll an even number at least once.

8. You toss a fair coin nine times and it lands tails up each time. The probability it will land heads up on the tenth flip is greater than 0.5.

9. A probability of $\frac{1}{10}$ indicates an unusual event.

10. If an event is almost certain to happen, its complement will be an unusual event.

Matching Probabilities *In Exercises 11–14, match the event with its probability.*

(a) 0.95 (b) 0.05 (c) 0.25 (d) 0

11. You toss a coin and randomly select a number from 0 to 9. What is the probability of getting tails and selecting a 3?

12. A random number generator is used to select a number from 1 to 100. What is the probability of selecting the number 153?

13. A game show contestant must randomly select a door. One door doubles her money while the other three doors leave her with no winnings. What is the probability she selects the door that doubles her money?

14. Five of the 100 digital video recorders (DVRs) in an inventory are known to be defective. What is the probability you randomly select an item that is not defective?

USING AND INTERPRETING CONCEPTS

Identifying a Sample Space *In Exercises 15–20, identify the sample space of the probability experiment and determine the number of outcomes in the sample space. Draw a tree diagram if it is appropriate.*

15. Guessing the initial of a student's middle name

16. Guessing a student's letter grade (A, B, C, D, F) in a class

17. Drawing one card from a standard deck of cards

18. Tossing three coins

19. Determining a person's blood type (A, B, AB, O) and Rh-factor (positive, negative)

20. Rolling a pair of six-sided dice

Recognizing Simple Events *In Exercises 21–24, determine the number of outcomes in each event. Then decide whether the event is a simple event or not. Explain your reasoning.*

21. A computer is used to randomly select a number between 1 and 4000. Event A is selecting 253.

22. A computer is used to randomly select a number between 1 and 4000. Event B is selecting a number less than 500.

23. You randomly select one card from a standard deck. Event A is selecting an ace.

24. You randomly select one card from a standard deck. Event B is selecting a ten of diamonds.

25. Job Openings A software company is hiring for two positions: a software development engineer and a sales operations manager. How many ways can these positions be filled if there are 12 people applying for the engineering position and 17 people applying for the managerial position?

26. Menu A restaurant offers a $12 dinner special that has 5 choices for an appetizer, 10 choices for entrées, and 4 choices for dessert. How many different meals are available if you select an appetizer, an entrée, and a dessert?

27. Realty A realtor uses a lock box to store the keys for a house that is for sale. The access code for the lock box consists of four digits. The first digit cannot be zero and the last digit must be even. How many different codes are available?

28. True or False Quiz Assuming that no questions are left unanswered, in how many ways can a six-question true-false quiz be answered?

Classical Probabilities *In Exercises 29–34, a probability experiment consists of rolling a 12-sided die. Find the probability of each event.*

29. Event A: rolling a 2

30. Event B: rolling a 10

31. Event C: rolling a number greater than 4

32. Event D: rolling an even number

33. Event E: rolling a prime number

34. Event F: rolling a number divisible by 5

Classifying Types of Probability *In Exercises 35 and 36, classify the statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.*

35. According to company records, the probability that a washing machine will need repairs during a six-year period is 0.10.

36. The probability of choosing 6 numbers from 1 to 40 that match the 6 numbers drawn by a state lottery is $1/3{,}838{,}380 \approx 0.00000026$.

Finding Probabilities *In Exercises 37–40, consider a company that selects employees for random drug tests. The company uses a computer to randomly select employee numbers that range from 1 to 6296.*

37. Find the probability of selecting a number less than 1000.

38. Find the probability of selecting a number greater than 1000.

39. Find the probability of selecting a number divisible by 1000.

40. Find the probability of selecting a number that is not divisible by 1000.

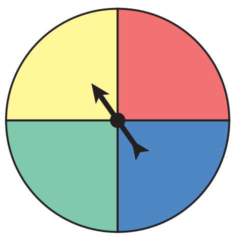


FIGURE FOR EXERCISES 41–44

Probability Experiment *In Exercises 41–44, a probability experiment consists of rolling a six-sided die and spinning the spinner shown at the left. The spinner is equally likely to land on each color. Use a tree diagram to find the probability of each event. Then tell whether the event can be considered unusual.*

41. Event A: rolling a 5 and the spinner landing on blue

42. Event B: rolling an odd number and the spinner landing on green

43. Event C: rolling a number less than 6 and the spinner landing on yellow

44. Event D: not rolling a number less than 6 and the spinner landing on yellow

45. Security System The access code for a garage door consists of three digits. Each digit can be any number from 0 through 9, and each digit can be repeated.

(a) Find the number of possible access codes.

(b) What is the probability of randomly selecting the correct access code on the first try?

(c) What is the probability of not selecting the correct access code on the first try?

46. Security System An access code consists of a letter followed by four digits. Any letter can be used, the first digit cannot be 0, and the last digit must be even.

(a) Find the number of possible access codes.

(b) What is the probability of randomly selecting the correct access code on the first try?

(c) What is the probability of not selecting the correct access code on the first try?

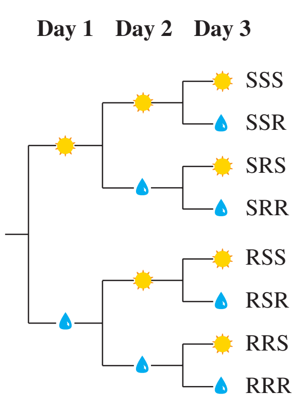


FIGURE FOR EXERCISES 47–50

Wet or Dry? *You are planning a three-day trip to Seattle, Washington in October. In Exercises 47–50, use the tree diagram shown at the left to answer each question.*

47. List the sample space.

48. List the outcome(s) of the event "It rains all three days."

49. List the outcome(s) of the event "It rains on exactly one day."

50. List the outcome(s) of the event "It rains on at least one day."

51. Sunny and Rainy Days You are planning a four-day trip to Seattle, Washington in October.

(a) Make a sunny day/rainy day tree diagram for your trip.

(b) List the sample space.

(c) List the outcome(s) of the event "It rains on exactly one day."

52. Machine Part Suppliers Your company buys machine parts from three different suppliers. Make a tree diagram that shows the three suppliers and whether the parts they supply are defective.

Graphical Analysis *In Exercises 53 and 54, use the diagram to answer the question.*

53. What is the probability that a registered voter in Virginia voted in the 2009 gubernatorial election? *(Source: Commonwealth of Virginia State Board of Elections)*

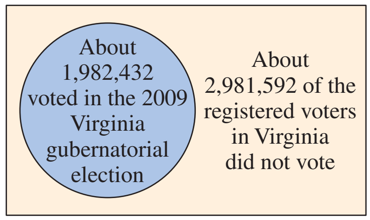


FIGURE FOR EXERCISE 53

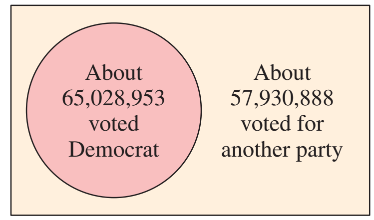


FIGURE FOR EXERCISE 54

54. What is the probability that a voter chosen at random did not vote for a Democratic representative in the 2008 election? *(Source: Federal Election Commission)*

Ages of voters	Frequency (in millions)
18 to 20	5.8
21 to 24	9.3
25 to 34	22.7
35 to 44	25.4
45 to 64	54.9
65 and over	28.1

TABLE FOR EXERCISES 55–58

Using a Frequency Distribution to Find Probabilities *In Exercises 55–58, use the frequency distribution at the left, which shows the number of American voters (in millions) according to age, to find the probability that a voter chosen at random is in the given age range. (Source: U.S. Census Bureau)*

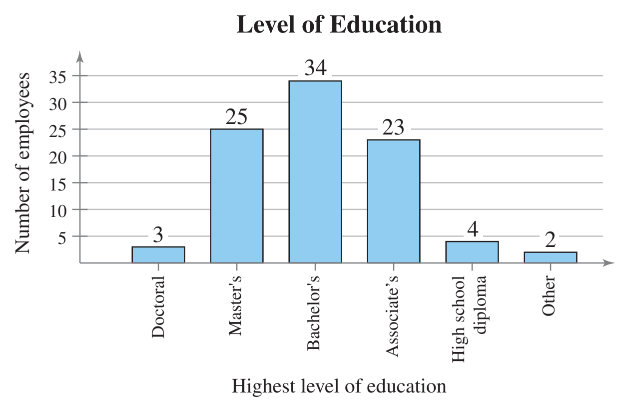
55. between 18 and 20 years old

56. between 35 and 44 years old

57. not between 21 and 24 years old

58. not between 45 and 64 years old

Using a Bar Graph to Find Probabilities *In Exercises 59–62, use the following bar graph, which shows the highest level of education received by employees of a company.*



Find the probability that the highest level of education for an employee chosen at random is

59. a doctorate.

60. an associate's degree.

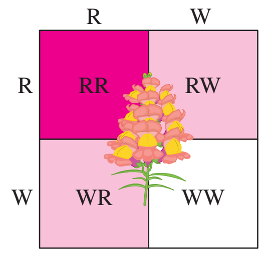
61. a master's degree.

62. a high school diploma.

63. Can any of the events in Exercises 55–58 be considered unusual? Explain.

64. Can any of the events in Exercises 59–62 be considered unusual? Explain.

65. Genetics A *Punnett square* is a diagram that shows all possible gene combinations in a cross of parents whose genes are known. When two pink snapdragon flowers (RW) are crossed, there are four equally likely possible outcomes for the genetic makeup of the offspring: red (RR), pink (RW), pink (WR), and white (WW), as shown in the Punnett square. If two pink snapdragons are crossed, what is the probability that the offspring will be (a) pink, (b) red, and (c) white?



Parents
Ssmm and SsMm

	SM	Sm
Sm	SSMm	SSmm
Sm	SSMm	SSmm
sm	SsMm	Ssmm
sm	SsMm	Ssmm

	sM	sm
Sm	SsMm	Ssmm
Sm	SsMm	Ssmm
sm	ssMm	ssmm
sm	ssMm	ssmm

TABLE FOR EXERCISE 66

66. Genetics There are six basic types of coloring in registered collies: sable (SSmm), tricolor (ssmm), trifactored sable (Ssmm), blue merle (ssMm), sable merle (SSMm), and trifactored sable merle (SsMm). The Punnett square at the left shows the possible coloring of the offspring of a trifactored sable merle collie and a trifactored sable collie. What is the probability that the offspring will have the same coloring as one of its parents?

Using a Pie Chart to Find Probabilities

In Exercises 67–70, use the pie chart at the left, which shows the number of workers (in thousands) by industry for the United States. *(Source: U.S. Bureau of Labor Statistics)*

Workers (in thousands) by Industry for the U.S.

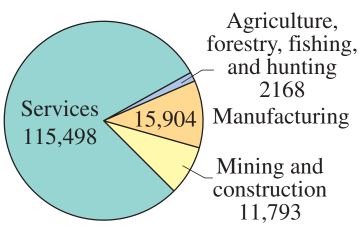


FIGURE FOR EXERCISES 67–70

67. Find the probability that a worker chosen at random was employed in the services industry.

68. Find the probability that a worker chosen at random was employed in the manufacturing industry.

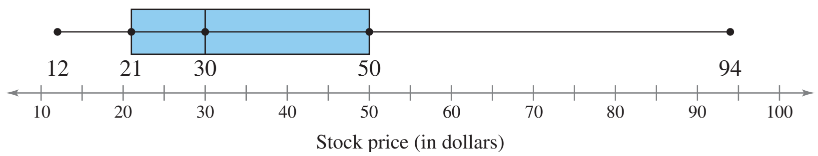
69. Find the probability that a worker chosen at random was not employed in the services industry.

70. Find the probability that a worker chosen at random was not employed in the agriculture, forestry, fishing, and hunting industry.

71. College Football A stem-and-leaf plot for the number of touchdowns scored by all NCAA Division I Football Bowl Subdivision teams is shown. If a team is selected at random, find the probability the team scored (a) at least 51 touchdowns, (b) between 20 and 30 touchdowns, inclusive, and (c) more than 69 touchdowns. Are any of these events unusual? Explain. *(Source: NCAA)*

```
1 | 8 8 9                      Key: 1|8 = 18
2 | 1 1 3 4 4 5 5 6 6 7 7 8 8 9 9
3 | 0 1 1 2 3 3 3 3 3 4 4 4 4 4 5 5 5 5 5 5 7 7 7 7 8 8 9 9 9
4 | 0 0 0 0 1 2 2 2 2 3 3 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 6 6 6 6 6 7 7 7 8 8 8 8 8 9 9 9
5 | 0 0 0 0 1 1 2 2 2 2 4 5 5 5 6 6 7 9
6 | 0 0 1 2 2 3 4 5 6 8 9
7 | 6 7
```

72. Individual Stock Price An individual stock is selected at random from the portfolio represented by the box-and-whisker plot shown. Find the probability that the stock price is (a) less than \$21, (b) between \$21 and \$50, and (c) \$30 or more.



Writing *In Exercises 73 and 74, write a statement that represents the complement of the given probability.*

73. The probability of randomly choosing a tea drinker who has a college degree (Assume that you are choosing from the population of all tea drinkers.)

74. The probability of randomly choosing a smoker whose mother also smoked (Assume that you are choosing from the population of all smokers.)

EXTENDING CONCEPTS

75. Rolling a Pair of Dice You roll a pair of six-sided dice and record the sum.

(a) List all of the possible sums and determine the probability of rolling each sum.

(b) Use a technology tool to simulate rolling a pair of dice and recording the sum 100 times. Make a tally of the 100 sums and use these results to list the probability of rolling each sum.

(c) Compare the probabilities in part (a) with the probabilities in part (b). Explain any similarities or differences.

Odds *In Exercises 76–81, use the following information. The chances of winning are often written in terms of odds rather than probabilities. The* ***odds of winning*** *is the ratio of the number of successful outcomes to the number of unsuccessful outcomes. The* ***odds of losing*** *is the ratio of the number of unsuccessful outcomes to the number of successful outcomes. For example, if the number of successful outcomes is 2 and the number of unsuccessful outcomes is 3, the odds of winning are 2 : 3 (read "2 to 3") or* $\frac{2}{3}$.

76. A beverage company puts game pieces under the caps of its drinks and claims that one in six game pieces wins a prize. The official rules of the contest state that the odds of winning a prize are $1:6$. Is the claim "one in six game pieces wins a prize" correct? Why or why not?

77. The probability of winning an instant prize game is $\frac{1}{10}$. The odds of winning a different instant prize game are $1:10$. If you want the best chance of winning, which game should you play? Explain your reasoning.

78. The odds of an event occurring are $4:5$. Find (a) the probability that the event will occur and (b) the probability that the event will not occur.

79. A card is picked at random from a standard deck of 52 playing cards. Find the odds that it is a spade.

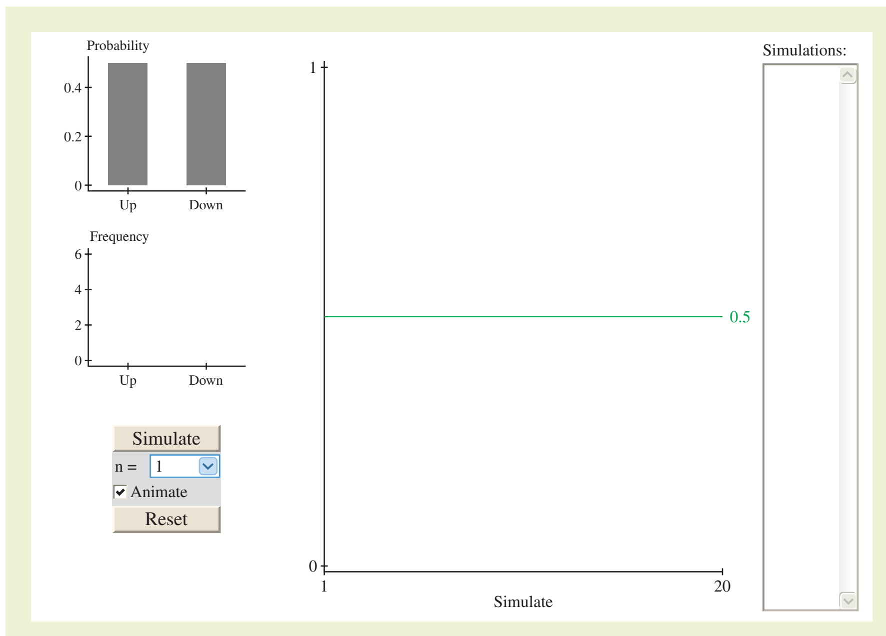
80. A card is picked at random from a standard deck of 52 playing cards. Find the odds that it is not a spade.

81. The odds of winning an event A are $p:q$. Show that the probability of event A is given by $P(A) = \dfrac{p}{p+q}$.

ACTIVITY 3.1 Simulating the Stock Market

APPLET

The *simulating the stock market* applet allows you to investigate the probability that the stock market will go up on any given day. The plot at the top left corner shows the probability associated with each outcome. In this case, the market has a 50% chance of going up on any given day. When SIMULATE is clicked, outcomes for n days are simulated. The results of the simulations are shown in the frequency plot. If the animate option is checked, the display will show each outcome dropping into the frequency plot as the simulation runs. The individual outcomes are shown in the text field at the far right of the applet. The center plot shows in red the cumulative proportion of times that the market went up. The green line in the plot reflects the true probability of the market going up. As the experiment is conducted over and over, the cumulative proportion should converge to the true value.



Explore

Step 1 Specify a value for n.
Step 2 Click SIMULATE four times.
Step 3 Click RESET.
Step 4 Specify another value for n.
Step 5 Click SIMULATE.

Draw Conclusions

APPLET

1. Run the simulation using $n = 1$ without clicking RESET. How many days did it take until there were three straight days on which the stock market went up? three straight days on which the stock market went down?

2. Run the applet to simulate the stock market activity over the last 35 business days. Find the empirical probability that the market goes up on day 36.

3.2 Conditional Probability and the Multiplication Rule

WHAT YOU SHOULD LEARN

- How to find the probability of an event given that another event has occurred
- How to distinguish between independent and dependent events
- How to use the Multiplication Rule to find the probability of two events occurring in sequence
- How to use the Multiplication Rule to find conditional probabilities

Conditional Probability ▸ Independent and Dependent Events ▸ The Multiplication Rule

▸ CONDITIONAL PROBABILITY

In this section, you will learn how to find the probability that two events occur in sequence. Before you can find this probability, however, you must know how to find *conditional probabilities*.

DEFINITION

A **conditional probability** is the probability of an event occurring, given that another event has already occurred. The conditional probability of event B occurring, given that event A has occurred, is denoted by $P(B|A)$ and is read as "probability of B, given A."

EXAMPLE 1

▸ Finding Conditional Probabilities

1. Two cards are selected in sequence from a standard deck. Find the probability that the second card is a queen, given that the first card is a king. (Assume that the king is not replaced.)
2. The table at the left shows the results of a study in which researchers examined a child's IQ and the presence of a specific gene in the child. Find the probability that a child has a high IQ, given that the child has the gene.

	Gene present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

▸ Solution

1. Because the first card is a king and is not replaced, the remaining deck has 51 cards, 4 of which are queens. So,

$$P(B|A) = \frac{4}{51} \approx 0.078.$$

So, the probability that the second card is a queen, given that the first card is a king, is about 0.078.

2. There are 72 children who have the gene. So, the sample space consists of these 72 children, as shown at the left. Of these, 33 have a high IQ. So,

$$P(B|A) = \frac{33}{72} \approx 0.458.$$

So, the probability that a child has a high IQ, given that the child has the gene, is about 0.458.

Sample Space

	Gene present
High IQ	33
Normal IQ	39
Total	72

▸ Try It Yourself 1

1. Find the probability that a child does not have the gene.
2. Find the probability that a child does not have the gene, given that the child has a normal IQ.

a. Find the *number of outcomes* in the event and in the sample space.
b. *Divide* the number of outcomes in the event by the number of outcomes in the sample space.

Answer: Page A35

PICTURING THE WORLD

Truman Collins, a probability and statistics enthusiast, wrote a program that finds the probability of landing on each square of a Monopoly board during a game. Collins explored various scenarios, including the effects of the Chance and Community Chest cards and the various ways of landing in or getting out of jail. Interestingly, Collins discovered that the length of each jail term affects the probabilities.

Monopoly square	Probability given short jail term	Probability given long jail term
Go	0.0310	0.0291
Chance	0.0087	0.0082
In Jail	0.0395	0.0946
Free Parking	0.0288	0.0283
Park Place	0.0219	0.0206
B&O RR	0.0307	0.0289
Water Works	0.0281	0.0265

Why do the probabilities depend on how long you stay in jail?

INDEPENDENT AND DEPENDENT EVENTS

In some experiments, one event does not affect the probability of another. For instance, if you roll a die and toss a coin, the outcome of the roll of the die does not affect the probability of the coin landing on heads. These two events are *independent*. The question of the independence of two or more events is important to researchers in fields such as marketing, medicine, and psychology. You can use conditional probabilities to determine whether events are *independent*.

DEFINITION

Two events are **independent** if the occurrence of one of the events does not affect the probability of the occurrence of the other event. Two events A and B are independent if

$$P(B|A) = P(B) \quad \text{or if} \quad P(A|B) = P(A).$$

Events that are not independent are **dependent.**

To determine if A and B are independent, first calculate $P(B)$, the probability of event B. Then calculate $P(B|A)$, the probability of B, given A. If the values are equal, the events are independent. If $P(B) \neq P(B|A)$, then A and B are dependent events.

EXAMPLE 2

Classifying Events as Independent or Dependent

Decide whether the events are independent or dependent.

1. Selecting a king from a standard deck (A), not replacing it, and then selecting a queen from the deck (B)
2. Tossing a coin and getting a head (A), and then rolling a six-sided die and obtaining a 6 (B)
3. Driving over 85 miles per hour (A), and then getting in a car accident (B)

Solution

1. $P(B|A) = \frac{4}{51}$ and $P(B) = \frac{4}{52}$. The occurrence of A changes the probability of the occurrence of B, so the events are dependent.
2. $P(B|A) = \frac{1}{6}$ and $P(B) = \frac{1}{6}$. The occurrence of A does not change the probability of the occurrence of B, so the events are independent.
3. If you drive over 85 miles per hour, the chances of getting in a car accident are greatly increased, so these events are dependent.

Try It Yourself 2

Decide whether the events are independent or dependent.

1. Smoking a pack of cigarettes per day (A) and developing emphysema, a chronic lung disease (B)
2. Exercising frequently (A) and having a 4.0 grade point average (B)

a. *Decide* whether the occurrence of the first event affects the probability of the second event.
b. State if the events are *independent* or *dependent*.

Answer: Page A35

THE MULTIPLICATION RULE

To find the probability of two events occurring in sequence, you can use the Multiplication Rule.

STUDY TIP

In words, to use the Multiplication Rule,

1. find the probability that the first event occurs,
2. find the probability that the second event occurs given that the first event has occurred, and
3. multiply these two probabilities.

THE MULTIPLICATION RULE FOR THE PROBABILITY OF *A* AND *B*

The probability that two events A and B will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B|A).$$

If events A and B are independent, then the rule can be simplified to $P(A \text{ and } B) = P(A) \cdot P(B)$. This simplified rule can be extended to any number of independent events.

EXAMPLE 3

Using the Multiplication Rule to Find Probabilities

1. Two cards are selected, without replacing the first card, from a standard deck. Find the probability of selecting a king and then selecting a queen.
2. A coin is tossed and a die is rolled. Find the probability of tossing a head and then rolling a 6.

Solution

1. Because the first card is not replaced, the events are dependent.

$$\begin{aligned} P(K \text{ and } Q) &= P(K) \cdot P(Q|K) \\ &= \frac{4}{52} \cdot \frac{4}{51} \\ &= \frac{16}{2652} \\ &\approx 0.006 \end{aligned}$$

So, the probability of selecting a king and then a queen is about 0.006.

2. The events are independent.

$$\begin{aligned} P(H \text{ and } 6) &= P(H) \cdot P(6) \\ &= \frac{1}{2} \cdot \frac{1}{6} \\ &= \frac{1}{12} \\ &\approx 0.083 \end{aligned}$$

So, the probability of tossing a head and then rolling a 6 is about 0.083.

Try It Yourself 3

1. The probability that a salmon swims successfully through a dam is 0.85. Find the probability that two salmon swim successfully through the dam.
2. Two cards are selected from a standard deck without replacement. Find the probability that they are both hearts.

a. Decide if the events are *independent* or *dependent*.
b. Use the *Multiplication Rule* to find the probability.

Answer: Page A35

EXAMPLE 4

Using the Multiplication Rule to Find Probabilities

The probability that a particular knee surgery is successful is 0.85.

1. Find the probability that three knee surgeries are successful.
2. Find the probability that none of the three knee surgeries are successful.
3. Find the probability that at least one of the three knee surgeries is successful.

Solution

1. The probability that each knee surgery is successful is 0.85. The chance of success for one surgery is independent of the chances for the other surgeries.

$$P(\text{three surgeries are successful}) = (0.85)(0.85)(0.85) \approx 0.614$$

So, the probability that all three surgeries are successful is about 0.614.

2. Because the probability of success for one surgery is 0.85, the probability of failure for one surgery is $1 - 0.85 = 0.15$.

$$P(\text{none of the three are successful}) = (0.15)(0.15)(0.15) \approx 0.003$$

So, the probability that none of the surgeries are successful is about 0.003. Because 0.003 is less than 0.05, this can be considered an unusual event.

3. The phrase "at least one" means one or more. The complement to the event "at least one is successful" is the event "none are successful." Use the complement to find the probability.

$$\begin{aligned} P(\text{at least one is successful}) &= 1 - P(\text{none are successful}) \\ &\approx 1 - 0.003 \\ &= 0.997. \end{aligned}$$

So, the probability that at least one of the three surgeries is successful is about 0.997.

Try It Yourself 4

The probability that a particular rotator cuff surgery is successful is 0.9. *(Source: The Orthopedic Center of St. Louis)*

1. Find the probability that three rotator cuff surgeries are successful.
2. Find the probability that none of the three rotator cuff surgeries are successful.
3. Find the probability that at least one of the three rotator cuff surgeries is successful.

a. Decide whether to find the probability of the event or its complement.
b. Use the *Multiplication Rule* to find the probability. If necessary, use the *complement*.
c. Determine if the event is *unusual*. Explain.

Answer: Page A35

In Example 4, you were asked to find a probability using the phrase "at least one." Notice that it was easier to find the probability of its complement, "none," and then subtract the probability of its complement from 1.

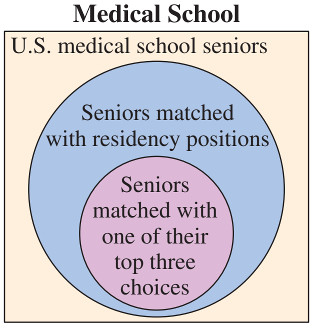
EXAMPLE 5

▸ Using the Multiplication Rule to Find Probabilities

More than 15,000 U.S. medical school seniors applied to residency programs in 2009. Of those, 93% were matched with residency positions. Eighty-two percent of the seniors matched with residency positions were matched with one of their top three choices. Medical students electronically rank the residency programs in their order of preference, and program directors across the United States do the same. The term "match" refers to the process whereby a student's preference list and a program director's preference list overlap, resulting in the placement of the student in a residency position. *(Source: National Resident Matching Program)*

1. Find the probability that a randomly selected senior was matched with a residency position *and* it was one of the senior's top three choices.
2. Find the probability that a randomly selected senior who was matched with a residency position did *not* get matched with one of the senior's top three choices.
3. Would it be unusual for a randomly selected senior to be matched with a residency position *and* that it was one of the senior's top three choices?

Medical School



▸ Solution

Let $A = \{\text{matched with residency position}\}$ and $B = \{\text{matched with one of top three choices}\}$. So, $P(A) = 0.93$ and $P(B|A) = 0.82$.

1. The events are dependent.

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = (0.93) \cdot (0.82) \approx 0.763$$

So, the probability that a randomly selected senior was matched with one of the senior's top three choices is about 0.763.

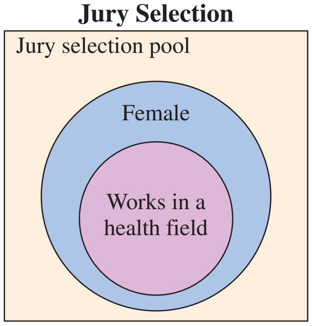
2. To find this probability, use the complement.

$$P(B'|A) = 1 - P(B|A) = 1 - 0.82 = 0.18$$

So, the probability that a randomly selected senior was matched with a residency position that was not one of the senior's top three choices is 0.18.

3. It is not unusual because the probability of a senior being matched with a residency position that was one of the senior's top three choices is about 0.763, which is greater than 0.05.

Jury Selection



▸ Try It Yourself 5

In a jury selection pool, 65% of the people are female. Of these 65%, one out of four works in a health field.

1. Find the probability that a randomly selected person from the jury pool is female and works in a health field.
2. Find the probability that a randomly selected person from the jury pool is female and does not work in a health field.

a. Determine *events* A and B.
b. Use the *Multiplication Rule* to write a formula to find the probability. If necessary, use the *complement*.
c. *Calculate* the probability.

Answer: Page A35

3.2 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. What is the difference between independent and dependent events?

2. List examples of

(a) two events that are independent.

(b) two events that are dependent.

3. What does the notation $P(B|A)$ mean?

4. Explain how the complement can be used to find the probability of getting at least one item of a particular type.

True or False? *In Exercises 5 and 6, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

5. If two events are independent, $P(A|B) = P(B)$.

6. If events A and B are dependent, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Classifying Events *In Exercises 7–12, decide whether the events are independent or dependent. Explain your reasoning.*

7. Selecting a king from a standard deck, replacing it, and then selecting a queen from the deck

8. Returning a rented movie after the due date and receiving a late fee

9. A father having hazel eyes and a daughter having hazel eyes

10. Not putting money in a parking meter and getting a parking ticket

11. Rolling a six-sided die and then rolling the die a second time so that the sum of the two rolls is five

12. A ball numbered from 1 through 52 is selected from a bin, replaced, and then a second numbered ball is selected from the bin.

Classifying Events Based on Studies *In Exercises 13–16, identify the two events described in the study. Do the results indicate that the events are independent or dependent? Explain your reasoning.*

13. A study found that people who suffer from moderate to severe sleep apnea are at increased risk of having high blood pressure. *(Source: Journal of the American Medical Association)*

14. Stress causes the body to produce higher amounts of acid, which can irritate already existing ulcers. But, stress does not cause stomach ulcers. *(Source: Baylor College of Medicine)*

15. Studies found that exposure to everyday sources of aluminum does not cause Alzheimer's disease. *(Source: Alzheimer's Association)*

16. According to researchers, diabetes is rare in societies in which obesity is rare. In societies in which obesity has been common for at least 20 years, diabetes is also common. *(Source: American Diabetes Association)*

USING AND INTERPRETING CONCEPTS

17. BRCA Gene In the general population, one woman in eight will develop breast cancer. Research has shown that approximately 1 woman in 600 carries a mutation of the BRCA gene. About 6 out of 10 women with this mutation develop breast cancer. *(Adapted from Susan G. Komen Breast Cancer Foundation)*

(a) Find the probability that a randomly selected woman will develop breast cancer, given that she has a mutation of the BRCA gene.

(b) Find the probability that a randomly selected woman will carry the mutation of the BRCA gene and will develop breast cancer.

(c) Are the events "carrying this mutation" and "developing breast cancer" independent or dependent? Explain.

Breast Cancer and the BRCA Gene

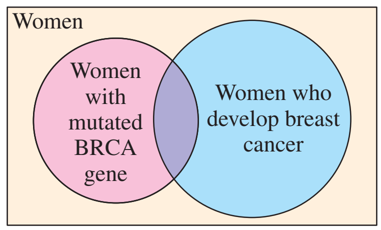


FIGURE FOR EXERCISE 17

What Do You Drive?

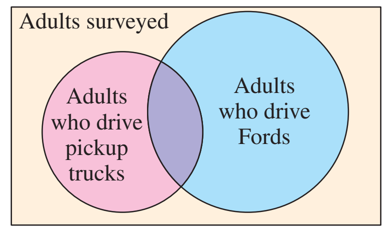


FIGURE FOR EXERCISE 18

18. Pickup Trucks In a survey, 510 adults were asked if they drive a pickup truck and if they drive a Ford. The results showed that one in six adults surveyed drives a pickup truck, and three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck.

(a) Find the probability that a randomly selected adult drives a pickup truck, given that the adult drives a Ford.

(b) Find the probability that a randomly selected adult drives a Ford and drives a pickup truck.

(c) Are the events "driving a Ford" and "driving a pickup truck" independent or dependent? Explain.

19. Summer Vacation The table shows the results of a survey in which 146 families were asked if they own a computer and if they will be taking a summer vacation during the current year.

		Summer Vacation This Year		
		Yes	**No**	**Total**
Own a Computer	**Yes**	87	28	115
	No	14	17	31
	Total	101	45	146

(a) Find the probability that a randomly selected family is not taking a summer vacation this year.

(b) Find the probability that a randomly selected family owns a computer.

(c) Find the probability that a randomly selected family is taking a summer vacation this year, given that they own a computer.

(d) Find the probability that a randomly selected family is taking a summer vacation this year and owns a computer.

(e) Are the events "owning a computer" and "taking a summer vacation this year" independent or dependent events? Explain.

20. Nursing Majors The table shows the number of male and female students enrolled in nursing at the University of Oklahoma Health Sciences Center for a recent semester. *(Source: University of Oklahoma Health Sciences Center Office of Institutional Research)*

	Nursing majors	Non-nursing majors	Total
Males	151	1104	1255
Females	1016	1693	2709
Total	1167	2797	3964

(a) Find the probability that a randomly selected student is a nursing major.

(b) Find the probability that a randomly selected student is male.

(c) Find the probability that a randomly selected student is a nursing major, given that the student is male.

(d) Find the probability that a randomly selected student is a nursing major and male.

(e) Are the events "being a male student" and "being a nursing major" independent or dependent events? Explain.

Pregnancies

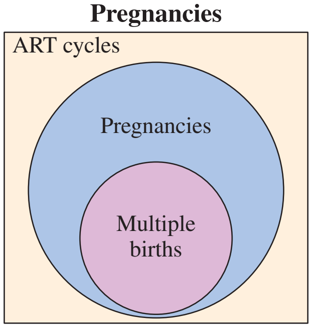


FIGURE FOR EXERCISE 21

21. Assisted Reproductive Technology A study found that 37% of the assisted reproductive technology (ART) cycles resulted in pregnancies. Twenty-five percent of the ART pregnancies resulted in multiple births. *(Source: National Center for Chronic Disease Prevention and Health Promotion)*

(a) Find the probability that a randomly selected ART cycle resulted in a pregnancy *and* produced a multiple birth.

(b) Find the probability that a randomly selected ART cycle that resulted in a pregnancy did *not* produce a multiple birth.

(c) Would it be unusual for a randomly selected ART cycle to result in a pregnancy and produce a multiple birth? Explain.

Government

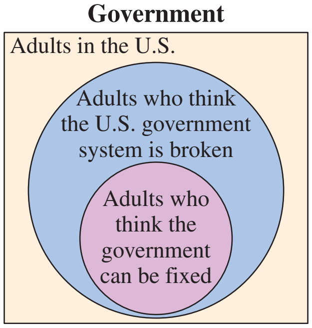


FIGURE FOR EXERCISE 22

22. Government According to a survey, 86% of adults in the United States think the U.S. government system is broken. Of these 86%, about 8 out of 10 think the government can be fixed. *(Adapted from CNN/Opinion Research Corporation)*

(a) Find the probability that a randomly selected adult thinks the U.S. government system is broken *and* thinks the government can be fixed.

(b) Given that a randomly selected adult thinks the U.S. government system is broken, find the probability that he or she thinks the government *cannot* be fixed.

(c) Would it be unusual for a randomly selected adult to think the U.S. government system is broken and think the government can be fixed? Explain.

23. Computers and Internet Access A study found that 81% of households in the United States have computers. Of those 81%, 92% have Internet access. Find the probability that a U.S. household selected at random has a computer and has Internet access. *(Source: The Nielsen Company)*

24. Surviving Surgery A doctor gives a patient a 60% chance of surviving bypass surgery after a heart attack. If the patient survives the surgery, he has a 50% chance that the heart damage will heal. Find the probability that the patient survives surgery and the heart damage heals.

25. People Who Can Wiggle Their Ears In a sample of 1000 people, 130 can wiggle their ears. Two unrelated people are selected at random without replacement.

(a) Find the probability that both people can wiggle their ears.

(b) Find the probability that neither person can wiggle his or her ears.

(c) Find the probability that at least one of the two people can wiggle his or her ears.

(d) Which of the events can be considered unusual? Explain.

26. Batteries Sixteen batteries are tested to see if they last as long as the manufacturer claims. Four batteries fail the test. Two batteries are selected at random without replacement.

(a) Find the probability that both batteries fail the test.

(b) Find the probability that both batteries pass the test.

(c) Find the probability that at least one battery fails the test.

(d) Which of the events can be considered unusual? Explain.

27. Emergency Savings The table shows the results of a survey in which 142 male and 145 female workers ages 25 to 64 were asked if they had at least one month's income set aside for emergencies.

	Male	Female	Total
Less than one month's income	66	83	149
One month's income or more	76	62	138
Total	142	145	287

(a) Find the probability that a randomly selected worker has one month's income or more set aside for emergencies.

(b) Given that a randomly selected worker is a male, find the probability that the worker has less than one month's income.

(c) Given that a randomly selected worker has one month's income or more, find the probability that the worker is a female.

(d) Are the events "having less than one month's income saved" and "being male" independent or dependent? Explain.

28. Health Care for Dogs The table shows the results of a survey in which 90 dog owners were asked how much they had spent in the last year for their dog's health care, and whether their dogs were purebred or mixed breeds.

		Type of Dog		
		Purebred	**Mixed breed**	**Total**
Health Care	**Less than $100**	19	21	40
	$100 or more	35	15	50
	Total	54	36	90

(a) Find the probability that $100 or more was spent on a randomly selected dog's health care in the last year.

(b) Given that a randomly selected dog owner spent less than $100, find the probability that the dog was a mixed breed.

(c) Find the probability that a randomly selected dog owner spent $100 or more on health care and the dog was a mixed breed.

(d) Are the events "spending $100 or more on health care" and "having a mixed breed dog" independent or dependent? Explain.

29. Blood Types The probability that a person in the United States has type B^+ blood is 9%. Five unrelated people in the United States are selected at random. *(Source: American Association of Blood Banks)*

(a) Find the probability that all five have type B^+ blood.
(b) Find the probability that none of the five have type B^+ blood.
(c) Find the probability that at least one of the five has type B^+ blood.

30. Blood Types The probability that a person in the United States has type A^+ blood is 31%. Three unrelated people in the United States are selected at random. *(Source: American Association of Blood Banks)*

(a) Find the probability that all three have type A^+ blood.
(b) Find the probability that none of the three have type A^+ blood.
(c) Find the probability that at least one of the three has type A^+ blood.

31. Guessing A multiple-choice quiz has five questions, each with four answer choices. Only one of the choices is correct. You have no idea what the answer is to any question and have to guess each answer.

(a) Find the probability of answering the first question correctly.
(b) Find the probability of answering the first two questions correctly.
(c) Find the probability of answering all five questions correctly.
(d) Find the probability of answering none of the questions correctly.
(e) Find the probability of answering at least one of the questions correctly.

32. Bookbinding Defects A printing company's bookbinding machine has a probability of 0.005 of producing a defective book. This machine is used to bind three books.

(a) Find the probability that none of the books are defective.
(b) Find the probability that at least one of the books is defective.
(c) Find the probability that all of the books are defective.

33. Warehouses A distribution center receives shipments of a product from three different factories in the following quantities: 50, 35, and 25. Three times a product is selected at random, each time without replacement. Find the probability that (a) all three products came from the third factory and (b) none of the three products came from the third factory.

34. Birthdays Three people are selected at random. Find the probability that (a) all three share the same birthday and (b) none of the three share the same birthday. Assume 365 days in a year.

EXTENDING CONCEPTS

According to ***Bayes' Theorem,*** *the probability of event A, given that event B has occurred, is*

$$P(A|B) = \frac{P(A)\cdot P(B|A)}{P(A)\cdot P(B|A) + P(A')\cdot P(B|A')}.$$

In Exercises 35–38, use Bayes' Theorem to find $P(A|B)$.

35. $P(A) = \frac{2}{3}$, $P(A') = \frac{1}{3}$, $P(B|A) = \frac{1}{5}$, and $P(B|A') = \frac{1}{2}$

36. $P(A) = \frac{3}{8}$, $P(A') = \frac{5}{8}$, $P(B|A) = \frac{2}{3}$, and $P(B|A') = \frac{3}{5}$

37. $P(A) = 0.25$, $P(A') = 0.75$, $P(B|A) = 0.3$, and $P(B|A') = 0.5$

38. $P(A) = 0.62$, $P(A') = 0.38$, $P(B|A) = 0.41$, and $P(B|A') = 0.17$

39. Reliability of Testing A certain virus infects one in every 200 people. A test used to detect the virus in a person is positive 80% of the time if the person has the virus and 5% of the time if the person does not have the virus. (This 5% result is called a *false positive*.) Let A be the event "the person is infected" and B be the event "the person tests positive."

(a) Using Bayes' Theorem, if a person tests positive, determine the probability that the person is infected.

(b) Using Bayes' Theorem, if a person tests negative, determine the probability that the person is *not* infected.

40. Birthday Problem You are in a class that has 24 students. You want to find the probability that at least two of the students share the same birthday.

(a) First, find the probability that each student has a different birthday.

$$P(\text{different birthdays}) = \overbrace{\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdots \frac{343}{365} \cdot \frac{342}{365}}^{\text{24 factors}}$$

(b) The probability that at least two students have the same birthday is the complement of the probability in part (a). What is this probability?

(c) We used a technology tool to generate 24 random numbers between 1 and 365. Each number represents a birthday. Did we get at least two people with the same birthday?

228	348	181	317	81	183
52	346	177	118	315	273
252	168	281	266	285	13
118	360	8	193	57	107

(d) Use a technology tool to simulate the "Birthday Problem." Repeat the simulation 10 times. How many times did you get at least two people with the same birthday?

The Multiplication Rule and Conditional Probability *By rewriting the formula for the Multiplication Rule, you can write a formula for finding conditional probabilities. The conditional probability of event B occurring, given that event A has occurred, is*

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}.$$

In Exercises 41 and 42, use the following information.

- *The probability that an airplane flight departs on time is 0.89.*
- *The probability that a flight arrives on time is 0.87.*
- *The probability that a flight departs and arrives on time is 0.83.*

41. Find the probability that a flight departed on time given that it arrives on time.

42. Find the probability that a flight arrives on time given that it departed on time.

3.3 The Addition Rule

WHAT YOU SHOULD LEARN

- How to determine if two events are mutually exclusive
- How to use the Addition Rule to find the probability of two events

Mutually Exclusive Events ▸ The Addition Rule ▸ A Summary of Probability

MUTUALLY EXCLUSIVE EVENTS

In Section 3.2, you learned how to find the probability of two events, A and B, occurring in sequence. Such probabilities are denoted by $P(A \text{ and } B)$. In this section, you will learn how to find the probability that at least one of two events will occur. Probabilities such as these are denoted by $P(A \text{ or } B)$ and depend on whether the events are *mutually exclusive*.

DEFINITION

Two events A and B are **mutually exclusive** if A and B cannot occur at the same time.

STUDY TIP

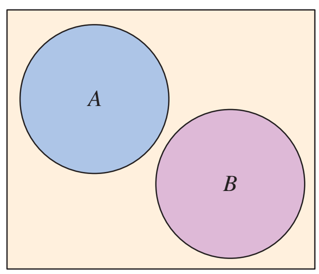
In probability and statistics, the word *or* is usually used as an "inclusive or" rather than an "exclusive or." For instance, there are three ways for "event *A* or *B*" to occur.

(1) *A* occurs and *B* does not occur.

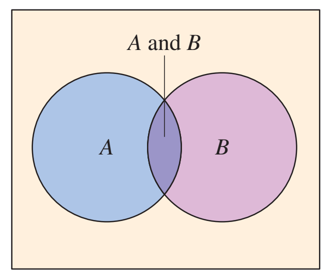
(2) *B* occurs and *A* does not occur.

(3) *A* and *B* both occur.

The Venn diagrams show the relationship between events that are mutually exclusive and events that are not mutually exclusive.



A and *B* are mutually exclusive.



A and *B* are not mutually exclusive.

EXAMPLE 1

▸ Mutually Exclusive Events

Decide if the events are mutually exclusive. Explain your reasoning.

1. Event A: Roll a 3 on a die.
Event B: Roll a 4 on a die.

2. Event A: Randomly select a male student.
Event B: Randomly select a nursing major.

3. Event A: Randomly select a blood donor with type O blood.
Event B: Randomly select a female blood donor.

▸ Solution

1. The first event has one outcome, a 3. The second event also has one outcome, a 4. These outcomes cannot occur at the same time, so the events are mutually exclusive.

2. Because the student can be a male nursing major, the events are not mutually exclusive.

3. Because the donor can be a female with type O blood, the events are not mutually exclusive.

Try It Yourself 1

Decide if the events are mutually exclusive. Explain your reasoning.

1. Event A: Randomly select a jack from a standard deck of cards.
Event B: Randomly select a face card from a standard deck of cards.

2. Event A: Randomly select a 20-year-old student.
Event B: Randomly select a student with blue eyes.

3. Event A: Randomly select a vehicle that is a Ford.
Event B: Randomly select a vehicle that is a Toyota.

a. Decide if one of the following statements is true.

- Events A and B cannot occur at the same time.
- Events A and B have no outcomes in common.
- $P(A \text{ and } B) = 0$

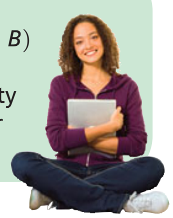
b. Make a *conclusion*.

Answer: Page A35

THE ADDITION RULE

STUDY TIP

By subtracting $P(A$ and $B)$ you avoid double counting the probability of outcomes that occur in both A and B.



To explore this topic further, see Activity 3.3 on page 166.

THE ADDITION RULE FOR THE PROBABILITY OF *A* OR *B*

The probability that events A or B will occur, $P(A \text{ or } B)$, is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

If events A and B are mutually exclusive, then the rule can be simplified to $P(A \text{ or } B) = P(A) + P(B)$. This simplified rule can be extended to any number of mutually exclusive events.

In words, to find the probability that one event or the other will occur, add the individual probabilities of each event and subtract the probability that they both occur.

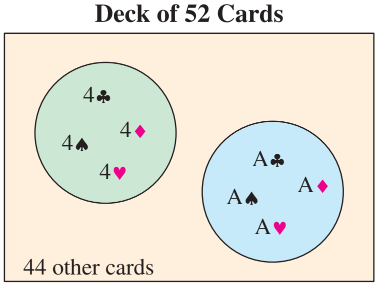
EXAMPLE 2

Using the Addition Rule to Find Probabilities

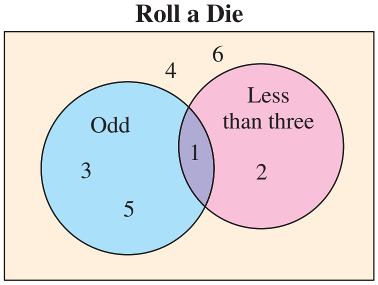
1. You select a card from a standard deck. Find the probability that the card is a 4 or an ace.

2. You roll a die. Find the probability of rolling a number less than 3 or rolling an odd number.

Deck of 52 Cards



Roll a Die



Solution

1. If the card is a 4, it cannot be an ace. So, the events are mutually exclusive, as shown in the Venn diagram. The probability of selecting a 4 or an ace is

$$P(4 \text{ or ace}) = P(4) + P(\text{ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \approx 0.154.$$

2. The events are not mutually exclusive because 1 is an outcome of both events, as shown in the Venn diagram. So, the probability of rolling a number less than 3 or an odd number is

$$\begin{aligned} P(\text{less than 3 or odd}) &= P(\text{less than 3}) + P(\text{odd}) \\ &\quad - P(\text{less than 3 and odd}) \\ &= \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \approx 0.667. \end{aligned}$$

▸ Try It Yourself 2

1. A die is rolled. Find the probability of rolling a 6 or an odd number.
2. A card is selected from a standard deck. Find the probability that the card is a face card or a heart.

a. Decide whether the events are *mutually exclusive*.
b. Find $P(A)$, $P(B)$, and, if necessary, $P(A \text{ and } B)$.
c. Use the *Addition Rule* to find the probability.

Answer: Page A35

EXAMPLE 3

▸ Finding Probabilities of Mutually Exclusive Events

The frequency distribution shows volumes of sales (in dollars) and the number of months in which a sales representative reached each sales level during the past three years. If this sales pattern continues, what is the probability that the sales representative will sell between \$75,000 and \$124,999 next month?

Sales volume ($)	Months
0–24,999	3
25,000–49,999	5
50,000–74,999	6
75,000–99,999	7
100,000–124,999	9
125,000–149,999	2
150,000–174,999	3
175,000–199,999	1

▸ Solution

To solve this problem, define events A and B as follows.

A = {monthly sales between \$75,000 and \$99,999}

B = {monthly sales between \$100,000 and \$124,999}

Because events A and B are mutually exclusive, the probability that the sales representative will sell between \$75,000 and \$124,999 next month is

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ &= \frac{7}{36} + \frac{9}{36} \\ &= \frac{16}{36} \\ &= \frac{4}{9} \approx 0.444. \end{aligned}$$

▸ Try It Yourself 3

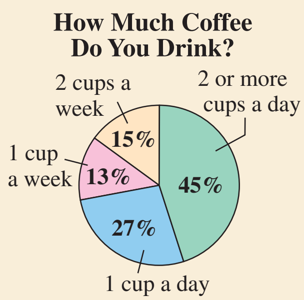
Find the probability that the sales representative will sell between \$0 and \$49,999.

a. Identify *events* A and B.
b. Decide if the events are *mutually exclusive*.
c. Find the *probability* of each event.
d. Use the *Addition Rule* to find the probability.

Answer: Page A35

PICTURING THE WORLD

In a survey conducted by Braun Research, coffee drinkers were asked how many cups of coffee they drink. (Source: Braun Research for International Delight Coffee House Inspirations)



If you selected a coffee drinker at random and asked how many cups of coffee he or she drinks, what is the probability that the coffee drinker would say he or she drinks 1 cup a week or 2 cups a week?

EXAMPLE 4

Using the Addition Rule to Find Probabilities

A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The number of donors who gave each blood type is shown in the table. A donor is selected at random.

1. Find the probability that the donor has type O or type A blood.
2. Find the probability that the donor has type B blood or is Rh-negative.

		Blood Type				
		O	**A**	**B**	**AB**	**Total**
Rh-factor	**Positive**	156	139	37	12	344
	Negative	28	25	8	4	65
	Total	184	164	45	16	409

Solution

1. Because a donor cannot have type O blood and type A blood, these events are mutually exclusive. So, using the Addition Rule, the probability that a randomly chosen donor has type O or type A blood is

$$\begin{aligned} P(\text{type O or type A}) &= P(\text{type O}) + P(\text{type A}) \\ &= \frac{184}{409} + \frac{164}{409} \\ &= \frac{348}{409} \\ &\approx 0.851. \end{aligned}$$

2. Because a donor can have type B blood and be Rh-negative, these events are not mutually exclusive. So, using the Addition Rule, the probability that a randomly chosen donor has type B blood or is Rh-negative is

$$\begin{aligned} P(\text{type B or Rh-neg}) &= P(\text{type B}) + P(\text{Rh-neg}) - P(\text{type B and Rh-neg}) \\ &= \frac{45}{409} + \frac{65}{409} - \frac{8}{409} \\ &= \frac{102}{409} \\ &\approx 0.249. \end{aligned}$$

Try It Yourself 4

1. Find the probability that the donor has type B or type AB blood.
2. Find the probability that the donor has type O blood or is Rh-positive.

a. Identify *events A* and *B*.
b. Decide if the events are *mutually exclusive*.
c. Find the *probability* of each event.
d. Use the *Addition Rule* to find the probability.

Answer: Page A35

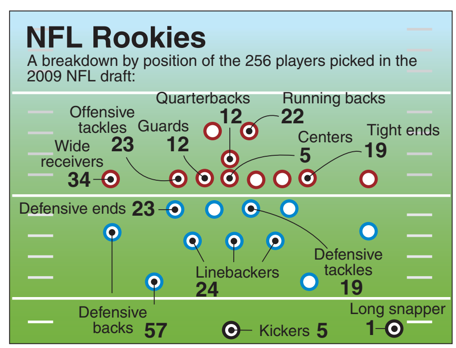
A SUMMARY OF PROBABILITY

Type of Probability and Probability Rules	In Words	In Symbols
Classical Probability	The number of outcomes in the sample space is known and each outcome is equally likely to occur.	$P(E) = \dfrac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$
Empirical Probability	The frequency of outcomes in the sample space is estimated from experimentation.	$P(E) = \dfrac{\text{Frequency of event } E}{\text{Total frequency}} = \dfrac{f}{n}$
Range of Probabilities Rule	The probability of an event is between 0 and 1, inclusive.	$0 \le P(E) \le 1$
Complementary Events	The complement of event E is the set of all outcomes in a sample space that are not included in E, denoted by E'.	$P(E') = 1 - P(E)$
Multiplication Rule	The Multiplication Rule is used to find the probability of two events occurring in a sequence.	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ *Independent events*
Addition Rule	The Addition Rule is used to find the probability of at least one of two events occurring.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ *Mutually exclusive events*

EXAMPLE 5

Combining Rules to Find Probabilities

Use the graph at the right to find the probability that a randomly selected draft pick is not a running back or a wide receiver.



(Source: National Football League)

Solution

Define events A and B.

A: Draft pick is a running back.
B: Draft pick is a wide receiver.

These events are mutually exclusive, so the probability that the draft pick is a running back or wide receiver is

$$P(A \text{ or } B) = P(A) + P(B) = \frac{22}{256} + \frac{34}{256} = \frac{56}{256} = \frac{7}{32} \approx 0.219.$$

By taking the complement of $P(A \text{ or } B)$, you can determine that the probability of randomly selecting a draft pick who is not a running back or wide receiver is

$$1 - P(A \text{ or } B) = 1 - \frac{7}{32} = \frac{25}{32} \approx 0.781.$$

Try It Yourself 5

Find the probability that a randomly selected draft pick is not a linebacker or a quarterback.

a. Find the *probability* that the draft pick is a linebacker or a quarterback.
b. Find the *complement* of the event. *Answer: Page A35*

3.3 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. If two events are mutually exclusive, why is $P(A \text{ and } B) = 0$?

2. List examples of

(a) two events that are mutually exclusive.
(b) two events that are not mutually exclusive.

True or False? *In Exercises 3–6, determine whether the statement is true or false. If it is false, explain why.*

3. If two events are mutually exclusive, they have no outcomes in common.

4. If two events are independent, then they are also mutually exclusive.

5. The probability that event A or event B will occur is

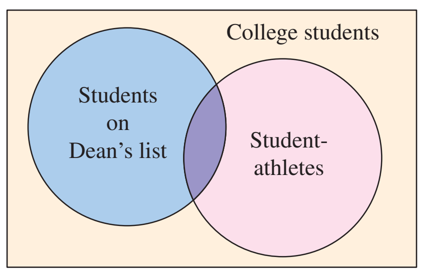
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ or } B).$$

6. If events A and B are mutually exclusive, then

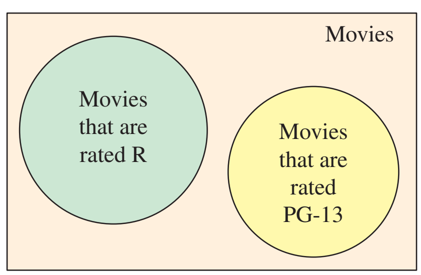
$$P(A \text{ or } B) = P(A) + P(B).$$

Graphical Analysis *In Exercises 7 and 8, decide if the events shown in the Venn diagram are mutually exclusive. Explain your reasoning.*

7.



8.



Recognizing Mutually Exclusive Events *In Exercises 9–12, decide if the events are mutually exclusive. Explain your reasoning.*

9. Event A: Randomly select a female public school teacher.
Event B: Randomly select a public school teacher who is 25 years old.

10. Event A: Randomly select a member of the U.S. Congress.
Event B: Randomly select a male U.S. Senator.

11. Event A: Randomly select a student with a birthday in April.
Event B: Randomly select a student with a birthday in May.

12. Event A: Randomly select a person between 18 and 24 years old.
Event B: Randomly select a person who drives a convertible.

USING AND INTERPRETING CONCEPTS

13. Audit During a 52-week period, a company paid overtime wages for 18 weeks and hired temporary help for 9 weeks. During 5 weeks, the company paid overtime *and* hired temporary help.

(a) Are the events "selecting a week in which overtime wages were paid" and "selecting a week in which temporary help wages were paid" mutually exclusive? Explain.

(b) If an auditor randomly examined the payroll records for only one week, what is the probability that the payroll for that week contained overtime wages or temporary help wages?

14. Conference A math conference has an attendance of 4950 people. Of these, 2110 are college professors and 2575 are female. Of the college professors, 960 are female.

(a) Are the events "selecting a female" and "selecting a college professor" mutually exclusive? Explain.

(b) The conference selects people at random to win prizes. Find the probability that a selected person is a female or a college professor.

15. Carton Defects A company that makes cartons finds that the probability of producing a carton with a puncture is 0.05, the probability that a carton has a smashed corner is 0.08, and the probability that a carton has a puncture and has a smashed corner is 0.004.

(a) Are the events "selecting a carton with a puncture" and "selecting a carton with a smashed corner" mutually exclusive? Explain.

(b) If a quality inspector randomly selects a carton, find the probability that the carton has a puncture or has a smashed corner.

16. Can Defects A company that makes soda pop cans finds that the probability of producing a can without a puncture is 0.96, the probability that a can does not have a smashed edge is 0.93, and the probability that a can does not have a puncture and does not have a smashed edge is 0.893.

(a) Are the events "selecting a can without a puncture" and "selecting a can without a smashed edge" mutually exclusive? Explain.

(b) If a quality inspector randomly selects a can, find the probability that the can does not have a puncture or does not have a smashed edge.

17. Selecting a Card A card is selected at random from a standard deck. Find each probability.

(a) Randomly selecting a club or a 3

(b) Randomly selecting a red suit or a king

(c) Randomly selecting a 9 or a face card

18. Rolling a Die You roll a die. Find each probability.

(a) Rolling a 5 or a number greater than 3

(b) Rolling a number less than 4 or an even number

(c) Rolling a 2 or an odd number

19. U.S. Age Distribution The estimated percent distribution of the U.S. population for 2020 is shown in the pie chart. Find each probability. *(Source: U.S. Census Bureau)*

(a) Randomly selecting someone who is under 5 years old
(b) Randomly selecting someone who is not 65 years or over
(c) Randomly selecting someone who is between 20 and 34 years old

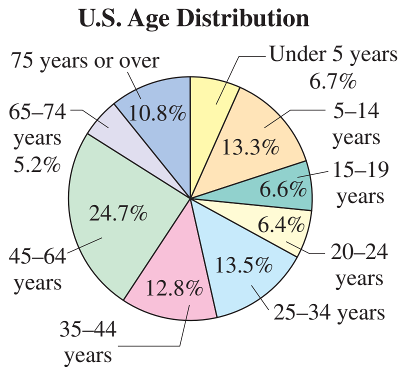


FIGURE FOR EXERCISE 19

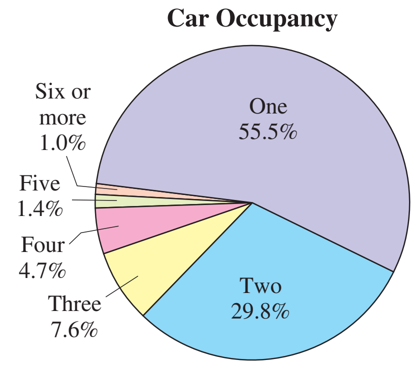


FIGURE FOR EXERCISE 20

20. Tacoma Narrows Bridge The percent distribution of the number of occupants in vehicles crossing the Tacoma Narrows Bridge in Washington is shown in the pie chart. Find each probability. *(Source: Washington State Department of Transportation)*

(a) Randomly selecting a car with two occupants
(b) Randomly selecting a car with two or more occupants
(c) Randomly selecting a car with between two and five occupants, inclusive

21. Education The number of responses to a survey are shown in the Pareto chart. The survey asked 1026 U.S. adults how they would grade the quality of public schools in the United States. Each person gave one response. Find each probability. *(Adapted from CBS News Poll)*

(a) Randomly selecting a person from the sample who did not give the public schools an A
(b) Randomly selecting a person from the sample who gave the public schools a D or an F

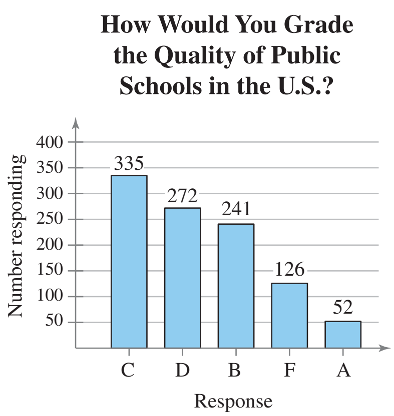


FIGURE FOR EXERCISE 21

22. Olympics The number of responses to a survey are shown in the Pareto chart. The survey asked 1000 U.S. adults if they would watch a large portion of the 2010 Winter Olympics. Each person gave one response. Find each probability. *(Adapted from Rasmussen Reports)*

(a) Randomly selecting a person from the sample who is not at all likely to watch a large portion of the Winter Olympics
(b) Randomly selecting a person from the sample who is not sure whether they will watch a large portion of the Winter Olympics
(c) Randomly selecting a person from the sample who is neither somewhat likely nor very likely to watch a large portion of the Winter Olympics

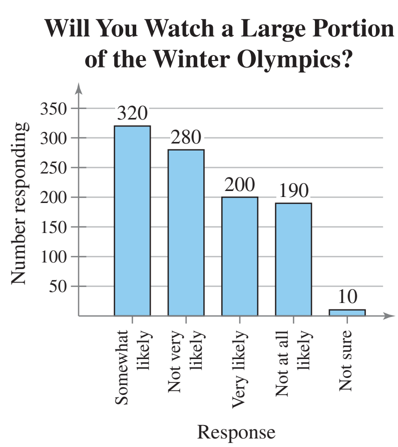


FIGURE FOR EXERCISE 22

23. Nursing Majors The table shows the number of male and female students enrolled in nursing at the University of Oklahoma Health Sciences Center for a recent semester. A student is selected at random. Find the probability of each event. *(Adapted from University of Oklahoma Health Sciences Center Office of Institutional Research)*

	Nursing majors	Non-nursing majors	Total
Males	151	1104	1255
Females	1016	1693	2709
Total	1167	2797	3964

(a) The student is male or a nursing major.

(b) The student is female or not a nursing major.

(c) The student is not female or is a nursing major.

(d) Are the events "being male" and "being a nursing major" mutually exclusive? Explain.

24. Left-Handed People In a sample of 1000 people (525 men and 475 women), 113 are left-handed (63 men and 50 women). The results of the sample are shown in the table. A person is selected at random from the sample. Find the probability of each event.

		Gender		
		Male	**Female**	**Total**
Dominant Hand	**Left**	63	50	113
	Right	462	425	887
	Total	525	475	1000

(a) The person is left-handed or female.

(b) The person is right-handed or male.

(c) The person is not right-handed or is a male.

(d) The person is right-handed and is a female.

(e) Are the events "being right-handed" and "being female" mutually exclusive? Explain.

25. Charity The table shows the results of a survey that asked 2850 people whether they were involved in any type of charity work. A person is selected at random from the sample. Find the probability of each event.

	Frequently	Occasionally	Not at all	Total
Male	221	456	795	1472
Female	207	430	741	1378
Total	428	886	1536	2850

(a) The person is frequently or occasionally involved in charity work.

(b) The person is female or not involved in charity work at all.

(c) The person is male or frequently involved in charity work.

(d) The person is female or not frequently involved in charity work.

(e) Are the events "being female" and "being frequently involved in charity work" mutually exclusive? Explain.

26. Eye Survey The table shows the results of a survey that asked 3203 people whether they wore contacts or glasses. A person is selected at random from the sample. Find the probability of each event.

	Only contacts	Only glasses	Both	Neither	Total
Male	64	841	177	456	1538
Female	189	427	368	681	1665
Total	253	1268	545	1137	3203

(a) The person wears only contacts or only glasses.

(b) The person is male or wears both contacts and glasses.

(c) The person is female or wears neither contacts nor glasses.

(d) The person is male or does not wear glasses.

(e) Are the events "wearing only contacts" and "wearing both contacts and glasses" mutually exclusive? Explain.

EXTENDING CONCEPTS

27. Writing Is there a relationship between independence and mutual exclusivity? To decide, find examples of the following, if possible.

(a) Describe two events that are dependent and mutually exclusive.

(b) Describe two events that are independent and mutually exclusive.

(c) Describe two events that are dependent and not mutually exclusive.

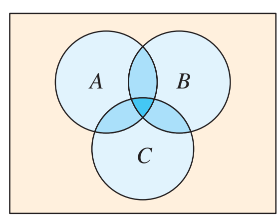
(d) Describe two events that are independent and not mutually exclusive.

Use your results to write a conclusion about the relationship between independence and mutual exclusivity.

Addition Rule for Three Events *The Addition Rule for the probability that event A or B or C will occur, $P(A \text{ or } B \text{ or } C)$, is given by*

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C).$$

In the Venn diagram shown, $P(A \text{ or } B \text{ or } C)$ is represented by the blue areas.



In Exercises 28 and 29, find $P(A \text{ or } B \text{ or } C)$ for the given probabilities.

28. $P(A) = 0.40$, $P(B) = 0.10$, $P(C) = 0.50$, $P(A \text{ and } B) = 0.05$, $P(A \text{ and } C) = 0.25$, $P(B \text{ and } C) = 0.10$, $P(A \text{ and } B \text{ and } C) = 0.03$

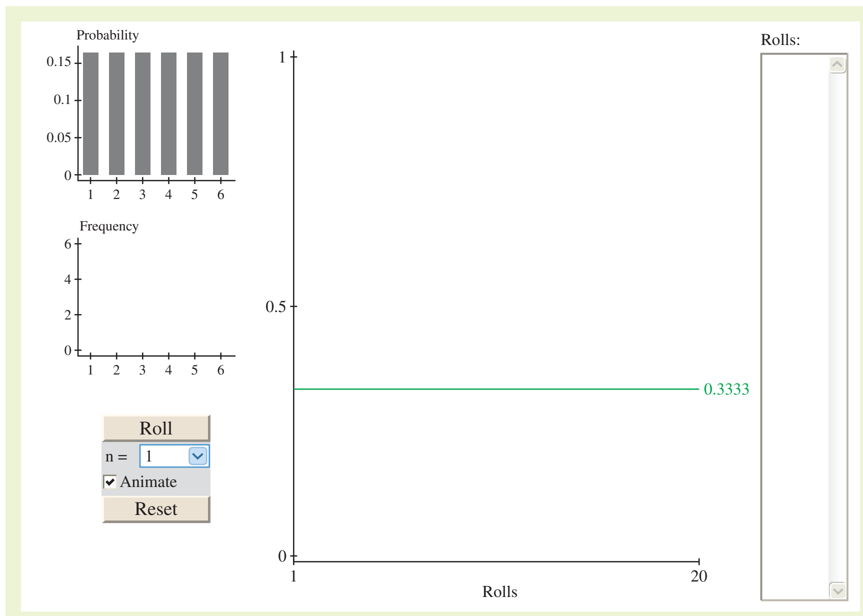
29. $P(A) = 0.38$, $P(B) = 0.26$, $P(C) = 0.14$, $P(A \text{ and } B) = 0.12$, $P(A \text{ and } C) = 0.03$, $P(B \text{ and } C) = 0.09$, $P(A \text{ and } B \text{ and } C) = 0.01$

30. Explain, in your own words, why in the Addition Rule for $P(A \text{ or } B \text{ or } C)$, $P(A \text{ and } B \text{ and } C)$ is added at the end of the formula.

ACTIVITY 3.3 Simulating the Probability of Rolling a 3 or 4

APPLET

The *simulating the probability of rolling a 3 or 4* applet allows you to investigate the probability of rolling a 3 or 4 on a fair die. The plot at the top left corner shows the probability associated with each outcome of a die roll. When ROLL is clicked, n simulations of the experiment of rolling a die are performed. The results of the simulations are shown in the frequency plot. If the animate option is checked, the display will show each outcome dropping into the frequency plot as the simulation runs. The individual outcomes are shown in the text field at the far right of the applet. The center plot shows in blue the cumulative proportion of times that an event of rolling a 3 or 4 occurs. The green line in the plot reflects the true probability of rolling a 3 or 4. As the experiment is conducted over and over, the cumulative proportion should converge to the true value.



Explore

Step 1 Specify a value for n.
Step 2 Click ROLL four times.
Step 3 Click RESET.
Step 4 Specify another value for n.
Step 5 Click ROLL.

Draw Conclusions

APPLET

1. What is the theoretical probability of rolling a 3 or 4?
2. Run the simulation using each value of n one time. Clear the results after each trial. Compare the cumulative proportion of rolling a 3 or 4 for each trial with the theoretical probability of rolling a 3 or 4.
3. Suppose you want to modify the applet so you can find the probability of rolling a number less than 4. Describe the placement of the green line.

United States Congress

Congress is made up of the House of Representatives and the Senate. Members of the House of Representatives serve two-year terms and represent a district in a state. The number of representatives each state has is determined by population. States with larger populations have more representatives than states with smaller populations. The total number of representatives is set by law at 435 members. Members of the Senate serve six-year terms and represent a state. Each state has 2 senators, for a total of 100. The tables show the makeup of the 111th Congress by gender and political party. There are two vacant seats in the House of Representatives.

House of Representatives

		Political Party			
		Republican	Democrat	Independent	Total
Gender	Male	161	196	0	357
	Female	17	59	0	76
	Total	178	255	0	433

Senate

		Political Party			
		Republican	Democrat	Independent	Total
Gender	Male	37	44	2	83
	Female	4	13	0	17
	Total	41	57	2	100

EXERCISES

1. Find the probability that a randomly selected representative is female. Find the probability that a randomly selected senator is female.

2. Compare the probabilities from Exercise 1.

3. A representative is selected at random. Find the probability of each event.

(a) The representative is male.

(b) The representative is a Republican.

(c) The representative is male given that the representative is a Republican.

(d) The representative is female and a Democrat.

(e) Are the events "being female" and "being a Democrat" independent or dependent events? Explain.

4. A senator is selected at random. Find the probability of each event.

(a) The senator is male.

(b) The senator is not a Democrat.

(c) The senator is female or a Republican.

(d) The senator is male or a Democrat.

(e) Are the events "being female" and "being an Independent" mutually exclusive? Explain.

5. Using the same row and column headings as the tables above, create a combined table for Congress.

6. A member of Congress is selected at random. Use the table from Exercise 5 to find the probability of each event.

(a) The member is Independent.

(b) The member is female and a Republican.

(c) The member is male or a Democrat.

3.4 Additional Topics in Probability and Counting

WHAT YOU SHOULD LEARN

- How to find the number of ways a group of objects can be arranged in order
- How to find the number of ways to choose several objects from a group without regard to order
- How to use counting principles to find probabilities

STUDY TIP

Notice at the right that as *n* increases, *n!* becomes very large. Take some time now to learn how to use the factorial key on your calculator.

Sudoku Number Puzzle

6	7	1				2	4	9
8			7		2			1
2				6				3
	5		6		3		2	
		8				7		
	1		8		4		6	
9				1				6
1			5		9			7
5	8	7				9	1	2

Permutations ▸ Combinations ▸ Applications of Counting Principles

PERMUTATIONS

In Section 3.1, you learned that the Fundamental Counting Principle is used to find the number of ways two or more events can occur in sequence. In this section, you will study several other techniques for counting the number of ways an event can occur. An important application of the Fundamental Counting Principle is determining the number of ways that n objects can be arranged in order or in a *permutation*.

DEFINITION

A **permutation** is an ordered arrangement of objects. The number of different permutations of n distinct objects is $n!$.

The expression $\boldsymbol{n!}$ is read as $\boldsymbol{n}$ **factorial** and is defined as follows.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 3 \cdot 2 \cdot 1$$

As a special case, $0! = 1$. Here are several other values of $n!$.

$$1! = 1,\ 2! = 2 \cdot 1 = 2,\ 3! = 3 \cdot 2 \cdot 1 = 6,\ 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

EXAMPLE 1

Finding the Number of Permutations of *n* Objects

The objective of a 9×9 Sudoku number puzzle is to fill the grid so that each row, each column, and each 3×3 grid contain the digits 1 to 9. How many different ways can the first row of a blank 9×9 Sudoku grid be filled?

Solution

The number of permutations is $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$. So, there are 362,880 different ways the first row can be filled.

Try It Yourself 1

The women's hockey teams for the 2010 Olympics are Canada, Sweden, Switzerland, Slovakia, United States, Finland, Russia, and China. How many different final standings are possible?

a. Determine the total number of women's hockey teams n that are in the 2010 Olympics.
b. Evaluate $n!$.

Answer: Page A35

Suppose you want to choose some of the objects in a group and put them in order. Such an ordering is called a **permutation of $\boldsymbol{n}$ objects taken $\boldsymbol{r}$ at a time.**

PERMUTATIONS OF *n* OBJECTS TAKEN *r* AT A TIME

The number of permutations of n distinct objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!}, \quad \text{where } r \leq n.$$

STUDY TIP

Detailed instructions for using MINITAB, Excel, and the TI-83/84 Plus are shown in the Technology Guide that accompanies this text. For instance, here are instructions for finding the number of permutations of n objects taken r at a time on a TI-83/84 Plus.

Enter the total number of objects n.

MATH

Choose the PRB menu.

2: nPr

Enter the number of objects r taken.

ENTER

EXAMPLE 2

Finding $_nP_r$

Find the number of ways of forming four-digit codes in which no digit is repeated.

Solution

To form a four-digit code with no repeating digits, you need to select 4 digits from a group of 10, so $n = 10$ and $r = 4$.

$$_nP_r = {}_{10}P_4 = \frac{10!}{(10-4)!}$$

$$= \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 5040$$

So, there are 5040 possible four-digit codes that do not have repeating digits.

Try It Yourself 2

A psychologist shows a list of eight activities to her subject. How many ways can the subject pick a first, second, and third activity?

a. Find the quotient of $n!$ and $(n - r)!$. (List the factors and divide out.)
b. *Write* the result as a sentence.

Answer: Page A35

INSIGHT

Notice that the Fundamental Counting Principle can be used in Example 3 to obtain the same result. There are 43 choices for first place, 42 choices for second place, and 41 choices for third place. So, there are

$$43 \cdot 42 \cdot 41 = 74{,}046$$

ways the cars can finish first, second, and third.

EXAMPLE 3

Finding $_nP_r$

Forty-three race cars started the 2010 Daytona 500. How many ways can the cars finish first, second, and third?

Solution

You need to select three race cars from a group of 43, so $n = 43$ and $r = 3$. Because the order is important, the number of ways the cars can finish first, second, and third is

$$_nP_r = {}_{43}P_3 = \frac{43!}{(43-3)!} = \frac{43!}{40!} = 43 \cdot 42 \cdot 41 = 74{,}046.$$

Try It Yourself 3

The board of directors of a company has 12 members. One member is the president, another is the vice president, another is the secretary, and another is the treasurer. How many ways can these positions be assigned?

a. *Identify* the total number of objects n and the number of objects r being chosen in order.
b. *Evaluate* $_nP_r$.

Answer: Page A35

STUDY TIP

The letters *AAAABBC* can be rearranged in 7! orders, but many of these are not distinguishable. The number of distinguishable orders is

$$\frac{7!}{4! \cdot 2! \cdot 1!} = \frac{7 \cdot 6 \cdot 5}{2}$$
$$= 105.$$

You may want to order a group of n objects in which some of the objects are the same. For instance, consider a group of letters consisting of four As, two Bs, and one C. How many ways can you order such a group? Using the previous formula, you might conclude that there are $_7P_7 = 7!$ possible orders. However, because some of the objects are the same, not all of these permutations are *distinguishable*. How many distinguishable permutations are possible? The answer can be found using the formula for the number of distinguishable permutations.

DISTINGUISHABLE PERMUTATIONS

The number of **distinguishable permutations** of n objects, where n_1 are of one type, n_2 are of another type, and so on, is

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}, \text{ where } n_1 + n_2 + n_3 + \cdots + n_k = n.$$

EXAMPLE 4

Finding the Number of Distinguishable Permutations

A building contractor is planning to develop a subdivision. The subdivision is to consist of 6 one-story houses, 4 two-story houses, and 2 split-level houses. In how many distinguishable ways can the houses be arranged?

Solution

There are to be 12 houses in the subdivision, 6 of which are of one type (one-story), 4 of another type (two-story), and 2 of a third type (split-level). So, there are

$$\frac{12!}{6! \cdot 4! \cdot 2!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4! \cdot 2!}$$
$$= 13{,}860 \text{ distinguishable ways.}$$

Interpretation There are 13,860 distinguishable ways to arrange the houses in the subdivision.

Try It Yourself 4

The contractor wants to plant six oak trees, nine maple trees, and five poplar trees along the subdivision street. The trees are to be spaced evenly. In how many distinguishable ways can they be planted?

a. *Identify* the total number of objects n and the number of each type of object in the groups n_1, n_2, and n_3.

b. *Evaluate* $\dfrac{n!}{n_1! \cdot n_2! \cdots n_k!}$.

Answer: Page A36

COMBINATIONS

You want to buy three DVDs from a selection of five DVDs labeled A, B, C, D, and E. There are 10 ways to make your selections.

ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

In each selection, order does not matter (*ABC* is the same set as *BAC*). The number of ways to choose r objects from n objects without regard to order is called the number of **combinations of n objects taken r at a time.**

INSIGHT

You can think of a combination of n objects chosen r at a time as a permutation of n objects in which the r selected objects are alike and the remaining $n - r$ (not selected) objects are alike.

COMBINATIONS OF n OBJECTS TAKEN r AT A TIME

A combination is a selection of r objects from a group of n objects without regard to order and is denoted by ${}_nC_r$. The number of combinations of r objects selected from a group of n objects is

$${}_nC_r = \frac{n!}{(n-r)!r!}.$$

STUDY TIP

Here are instructions for finding the number of combinations of n objects taken r at a time on a TI-83/84 Plus.

Enter the total number of objects n.

MATH

Choose the PRB menu.

3: nCr

Enter the number of objects r taken.

ENTER

EXAMPLE 5

Finding the Number of Combinations

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

Solution

The state is selecting four companies from a group of 16, so $n = 16$ and $r = 4$. Because order is not important, there are

$$ {}_nC_r = {}_{16}C_4 = \frac{16!}{(16-4)!4!} $$
$$ = \frac{16!}{12!4!} $$
$$ = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4!} $$
$$ = 1820 \text{ different combinations.} $$

Interpretation There are 1820 different combinations of four companies that can be selected from the 16 bidding companies.

Try It Yourself 5

The manager of an accounting department wants to form a three-person advisory committee from the 20 employees in the department. In how many ways can the manager form this committee?

a. *Identify* the number of objects in the group n and the number of objects r to be selected.
b. *Evaluate* ${}_nC_r$.
c. *Write* the result as a sentence.

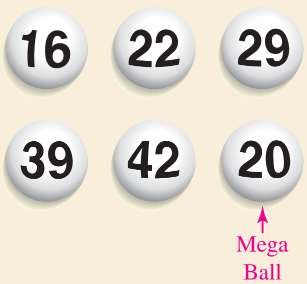
Answer: Page A36

The table summarizes the counting principles.

Principle	Description	Formula
Fundamental Counting Principle	If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$.	$m \cdot n$
Permutations	The number of different ordered arrangements of n distinct objects	$n!$
	The number of permutations of n distinct objects taken r at a time, where $r \leq n$	${}_nP_r = \frac{n!}{(n-r)!}$
	The number of distinguishable permutations of n objects where n_1 are of one type, n_2 are of another type, and so on	$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$
Combinations	The number of combinations of r objects selected from a group of n objects without regard to order	${}_nC_r = \frac{n!}{(n-r)!r!}$

PICTURING THE WORLD

The largest lottery jackpot ever, \$390 million, was won in the Mega Millions lottery. When the Mega Millions jackpot was won, five numbers were chosen from 1 to 56 and one number, the Mega Ball, was chosen from 1 to 46. The winning numbers are shown below.



If you buy one ticket, what is the probability that you will win the Mega Millions lottery?

APPLICATIONS OF COUNTING PRINCIPLES

EXAMPLE 6

Finding Probabilities

A student advisory board consists of 17 members. Three members serve as the board's chair, secretary, and webmaster. Each member is equally likely to serve in any of the positions. What is the probability of selecting at random the three members who currently hold the three positions?

Solution There is one favorable outcome and there are

$$_{17}P_3 = \frac{17!}{(17-3)!} = \frac{17!}{14!} = \frac{17 \cdot 16 \cdot 15 \cdot 14!}{14!} = 17 \cdot 16 \cdot 15 = 4080$$

ways the three positions can be filled. So, the probability of correctly selecting the three members who hold each position is

$$P(\text{selecting the three members}) = \frac{1}{4080} \approx 0.0002.$$

Try It Yourself 6

A student advisory board consists of 20 members. Two members serve as the board's chair and secretary. Each member is equally likely to serve in either of the positions. What is the probability of selecting at random the two members who currently hold the two positions?

a. Find the *number of ways* the two positions can be filled.
b. Find the *probability* of correctly selecting the two members.

Answer: Page A36

EXAMPLE 7

Finding Probabilities

You have 11 letters consisting of one M, four I's, four S's, and two P's. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word *Mississippi*?

Solution There is one favorable outcome and there are

$$\frac{11!}{1! \cdot 4! \cdot 4! \cdot 2!} = 34{,}650$$ 11 letters with 1, 4, 4, and 2 like letters

distinguishable permutations of the given letters. So, the probability that the arrangement spells the word *Mississippi* is

$$P(\text{Mississippi}) = \frac{1}{34{,}650} \approx 0.00003.$$

Try It Yourself 7

You have 6 letters consisting of one L, two E's, two T's, and one R. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word *letter*?

a. Find the *number of favorable outcomes* and the *number of distinguishable permutations*.
b. Find the *probability* that the arrangement spells the word *letter*.

Answer: Page A36

EXAMPLE 8

Finding Probabilities

Find the probability of picking five diamonds from a standard deck of playing cards.

Solution

The possible number of ways of choosing 5 diamonds out of 13 is ${}_{13}C_5$. The number of possible five-card hands is ${}_{52}C_5$. So, the probability of being dealt 5 diamonds is

$$P(5 \text{ diamonds}) = \frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2{,}598{,}960} \approx 0.0005.$$

Try It Yourself 8

Find the probability of being dealt five diamonds from a standard deck of playing cards that also includes two jokers. In this case, the joker is considered to be a wild card that can be used to represent any card in the deck.

a. Find the *number of ways* of choosing 5 diamonds.
b. Find the *number of possible five-card hands*.
c. Find the *probability* of being dealt five diamonds. *Answer: Page A36*

EXAMPLE 9

Finding Probabilities

A food manufacturer is analyzing a sample of 400 corn kernels for the presence of a toxin. In this sample, three kernels have dangerously high levels of the toxin. If four kernels are randomly selected from the sample, what is the probability that exactly one kernel contains a dangerously high level of the toxin?

Solution

The possible number of ways of choosing one toxic kernel out of three toxic kernels is ${}_3C_1$. The possible number of ways of choosing 3 nontoxic kernels from 397 nontoxic kernels is ${}_{397}C_3$. So, using the Fundamental Counting Principle, the number of ways of choosing one toxic kernel and three nontoxic kernels is

$$\begin{aligned} {}_3C_1 \cdot {}_{397}C_3 &= 3 \cdot 10{,}349{,}790 \\ &= 31{,}049{,}370. \end{aligned}$$

The number of possible ways of choosing 4 kernels from 400 kernels is ${}_{400}C_4 = 1{,}050{,}739{,}900$. So, the probability of selecting exactly 1 toxic kernel is

$$P(1 \text{ toxic kernel}) = \frac{{}_3C_1 \cdot {}_{397}C_3}{{}_{400}C_4} = \frac{31{,}049{,}370}{1{,}050{,}739{,}900} \approx 0.030.$$

Try It Yourself 9

A jury consists of five men and seven women. Three jury members are selected at random for an interview. Find the probability that all three are men.

a. *Find* the product of the number of ways to choose three men from five and the number of ways to choose zero women from seven.
b. Find the *number of ways* to choose 3 jury members from 12.
c. Find the *probability* that all three are men. *Answer: Page A36*

3.4 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. When you calculate the number of permutations of n distinct objects taken r at a time, what are you counting? Give an example.

2. When you calculate the number of combinations of r objects taken from a group of n objects, what are you counting? Give an example.

True or False? *In Exercises 3–6, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

3. A combination is an ordered arrangement of objects.

4. The number of different ordered arrangements of n distinct objects is $n!$.

5. If you divide the number of permutations of 11 objects taken 3 at a time by 3!, you will get the number of combinations of 11 objects taken 3 at a time.

6. ${}_7C_5 = {}_7C_2$

In Exercises 7–14, perform the indicated calculation.

7. ${}_9P_5$

8. ${}_{16}P_2$

9. ${}_8C_3$

10. ${}_7P_4$

11. ${}_{21}C_8$

12. $\dfrac{{}_8C_4}{{}_{12}C_6}$

13. $\dfrac{{}_6P_2}{{}_{11}P_3}$

14. $\dfrac{{}_{10}C_7}{{}_{14}C_7}$

In Exercises 15–18, decide if the situation involves permutations, combinations, or neither. Explain your reasoning.

15. The number of ways eight cars can line up in a row for a car wash

16. The number of ways a four-member committee can be chosen from 10 people

17. The number of ways 2 captains can be chosen from 28 players on a lacrosse team

18. The number of four-letter passwords that can be created when no letter can be repeated

USING AND INTERPRETING CONCEPTS

19. Video Games You have seven different video games. How many different ways can you arrange the games side by side on a shelf?

20. Skiing Eight people compete in a downhill ski race. Assuming that there are no ties, in how many different orders can the skiers finish?

21. Security Code In how many ways can the letters A, B, C, D, E, and F be arranged for a six-letter security code?

22. Starting Lineup The starting lineup for a softball team consists of 10 players. How many different batting orders are possible using the starting lineup?

23. Lottery Number Selection A lottery has 52 numbers. In how many different ways can 6 of the numbers be selected? (Assume that order of selection is not important.)

24. Assembly Process There are four processes involved in assembling a certain product. These processes can be performed in any order. Management wants to find which order is the least time-consuming. How many different orders will have to be tested?

25. Bracelets You are putting 4 spacers, 10 gold charms, and 8 silver charms on a bracelet. In how many distinguishable ways can the spacers and charms be put on the bracelet?

26. Experimental Group In order to conduct an experiment, 4 subjects are randomly selected from a group of 20 subjects. How many different groups of four subjects are possible?

27. Letters In how many distinguishable ways can the letters in the word *statistics* be written?

28. Jury Selection From a group of 40 people, a jury of 12 people is selected. In how many different ways can a jury of 12 people be selected?

29. Space Shuttle Menu Space shuttle astronauts each consume an average of 3000 calories per day. One meal normally consists of a main dish, a vegetable dish, and two different desserts. The astronauts can choose from 10 main dishes, 8 vegetable dishes, and 13 desserts. How many different meals are possible? *(Source: NASA)*

30. Menu A restaurant offers a dinner special that has 12 choices for entrées, 10 choices for side dishes, and 6 choices for dessert. For the special, you can choose one entrée, two side dishes, and one dessert. How many different meals are possible?

31. Water Samples An environmental agency is analyzing water samples from 80 lakes for pollution. Five of the lakes have dangerously high levels of dioxin. If six lakes are randomly selected from the sample, how many ways could one polluted lake and five non-polluted lakes be chosen? Use a technology tool.

32. Soil Samples An environmental agency is analyzing soil samples from 50 farms for lead contamination. Eight of the farms have dangerously high levels of lead. If 10 farms are randomly selected from the sample, how many ways could 2 contaminated farms and 8 noncontaminated farms be chosen? Use a technology tool.

Word Jumble *In Exercises 33–38, do the following.*

(a) Find the number of distinguishable ways the letters can be arranged.

(b) There is one arrangement that spells an important term used throughout the course. Find the term.

(c) If the letters are randomly arranged in order, what is the probability that the arrangement spells the word from part (b)? Can this event be considered unusual? Explain.

33. palmes

34. nevte

35. etre

36. rnctee

37. unoppolati

38. sidtbitoiurn

39. Horse Race A horse race has 12 entries. Assuming that there are no ties, what is the probability that the three horses owned by one person finish first, second, and third?

40. Pizza Toppings A pizza shop offers nine toppings. No topping is used more than once. What is the probability that the toppings on a three-topping pizza are pepperoni, onions, and mushrooms?

41. Jukebox You look over the songs on a jukebox and determine that you like 15 of the 56 songs.

(a) What is the probability that you like the next three songs that are played? (Assume a song cannot be repeated.)

(b) What is the probability that you do not like the next three songs that are played? (Assume a song cannot be repeated.)

42. Officers The offices of president, vice president, secretary, and treasurer for an environmental club will be filled from a pool of 14 candidates. Six of the candidates are members of the debate team.

(a) What is the probability that all of the offices are filled by members of the debate team?

(b) What is the probability that none of the offices are filled by members of the debate team?

43. Employee Selection Four sales representatives for a company are to be chosen to participate in a training program. The company has eight sales representatives, two in each of four regions. In how many ways can the four sales representatives be chosen if (a) there are no restrictions and (b) the selection must include a sales representative from each region? (c) What is the probability that the four sales representatives chosen to participate in the training program will be from only two of the four regions if they are chosen at random?

44. License Plates In a certain state, each automobile license plate number consists of two letters followed by a four-digit number. How many distinct license plate numbers can be formed if (a) there are no restrictions and (b) the letters O and I are not used? (c) What is the probability of selecting at random a license plate that ends in an even number?

45. Password A password consists of two letters followed by a five-digit number. How many passwords are possible if (a) there are no restrictions and (b) none of the letters or digits can be repeated? (c) What is the probability of guessing the password in one trial if there are no restrictions?

46. Area Code An area code consists of three digits. How many area codes are possible if (a) there are no restrictions and (b) the first digit cannot be a 1 or a 0? (c) What is the probability of selecting an area code at random that ends in an odd number if the first digit cannot be a 1 or a 0?

47. Repairs In how many orders can three broken computers and two broken printers be repaired if (a) there are no restrictions, (b) the printers must be repaired first, and (c) the computers must be repaired first? (d) If the order of repairs has no restrictions and the order of repairs is done at random, what is the probability that a printer will be repaired first?

48. Defective Units A shipment of 10 microwave ovens contains two defective units. In how many ways can a restaurant buy three of these units and receive (a) no defective units, (b) one defective unit, and (c) at least two nondefective units? (d) What is the probability of the restaurant buying at least two nondefective units?

Rate Your Financial Shape

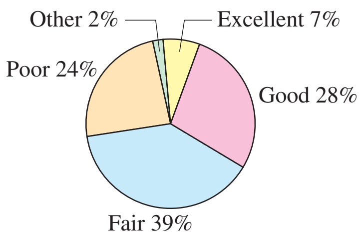


FIGURE FOR EXERCISES 49–52

Financial Shape *In Exercises 49–52, use the pie chart, which shows how U.S. adults rate their financial shape.* *(Source: Pew Research Center)*

49. Suppose 4 people are chosen at random from a group of 1200. What is the probability that all four would rate their financial shape as excellent? (Make the assumption that the 1200 people are represented by the pie chart.)

50. Suppose 10 people are chosen at random from a group of 1200. What is the probability that all 10 would rate their financial shape as poor? (Make the assumption that the 1200 people are represented by the pie chart.)

51. Suppose 80 people are chosen at random from a group of 500. What is the probability that none of the 80 people would rate their financial shape as fair? (Make the assumption that the 500 people are represented by the pie chart.)

52. Suppose 55 people are chosen at random from a group of 500. What is the probability that none of the 55 people would rate their financial shape as good? (Make the assumption that the 500 people are represented by the pie chart.)

53. Probability In a state lottery, you must correctly select 5 numbers (in any order) out of 40 to win the top prize.

(a) How many ways can 5 numbers be chosen from 40 numbers?

(b) You purchase one lottery ticket. What is the probability that you will win the top prize?

54. Probability A company that has 200 employees chooses a committee of 15 to represent employee retirement issues. When the committee is formed, none of the 56 minority employees are selected.

(a) Use a technology tool to find the number of ways 15 employees can be chosen from 200.

(b) Use a technology tool to find the number of ways 15 employees can be chosen from 144 nonminorities.

(c) If the committee is chosen randomly (without bias), what is the probability that it contains no minorities?

(d) Does your answer to part (c) indicate that the committee selection is biased? Explain your reasoning.

55. Cards You are dealt a hand of five cards from a standard deck of playing cards. Find the probability of being dealt a hand consisting of

(a) four-of-a-kind.

(b) a full house, which consists of three of one kind and two of another kind.

(c) three-of-a-kind. (The other two cards are different from each other.)

(d) two clubs and one of each of the other three suits.

56. Warehouse A warehouse employs 24 workers on first shift and 17 workers on second shift. Eight workers are chosen at random to be interviewed about the work environment. Find the probability of choosing

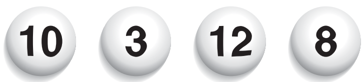
(a) all first-shift workers.

(b) all second-shift workers.

(c) six first-shift workers.

(d) four second-shift workers.

EXTENDING CONCEPTS



NBA Draft Lottery *In Exercises 57–62, use the following information. The National Basketball Association (NBA) uses a lottery to determine which team gets the first pick in its annual draft. The teams eligible for the lottery are the 14 non-playoff teams. Fourteen Ping-Pong balls numbered 1 through 14 are placed in a drum. Each of the 14 teams is assigned a certain number of possible four-number combinations that correspond to the numbers on the Ping-Pong balls, such as 3, 8, 10, and 12, as shown. Four balls are then drawn out to determine the first pick in the draft. The order in which the balls are drawn is not important. All of the four-number combinations are assigned to the 14 teams by computer except for one four-number combination. When this four-number combination is drawn, the balls are put back in the drum and another drawing takes place. For instance, if Team A has been assigned the four-number combination 3, 8, 10, 12 and the balls shown at the left are drawn, then Team A wins the first pick.*

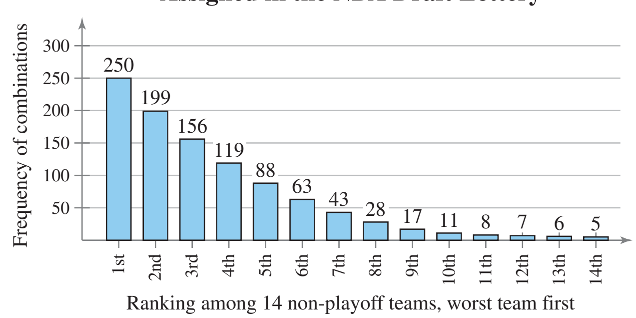
After the first pick of the draft is determined, the process continues to choose the teams that will select second and third picks. A team may not win the lottery more than once. If the four-number combination belonging to a team that has already won is drawn, the balls are put back in the drum and another drawing takes place. The remaining order of the draft is determined by the number of losses of each team.

57. In how many ways can 4 of the numbers 1 to 14 be selected if order is not important? How many sets of 4 numbers are assigned to the 14 teams?

58. In how many ways can four of the numbers be selected if order is important?

In the Pareto chart, the number of combinations assigned to each of the 14 teams is shown. The team with the most losses (the worst team) gets the most chances to win the lottery. So, the worst team receives the greatest frequency of four-number combinations, 250. The team with the best record of the 14 non-playoff teams has the fewest chances, with 5 four-number combinations.

Frequency of Four-Number Combinations Assigned in the NBA Draft Lottery



59. For each team, find the probability that the team will win the first pick. Which of these events would be considered unusual? Explain.

60. What is the probability that the team with the worst record will win the second pick, given that the team with the best record, ranked 14th, wins the first pick?

61. What is the probability that the team with the worst record will win the third pick, given that the team with the best record, ranked 14th, wins the first pick and the team ranked 2nd wins the second pick?

62. What is the probability that neither the first- nor the second-worst team will get the first pick?

USES AND ABUSES

Uses

Probability affects decisions when the weather is forecast, when marketing strategies are determined, when medications are selected, and even when players are selected for professional sports teams. Although intuition is often used for determining probabilities, you will be better able to assess the likelihood that an event will occur by applying the rules of classical probability and empirical probability.

For instance, suppose you work for a real estate company and are asked to estimate the likelihood that a particular house will sell for a particular price within the next 90 days. You could use your intuition, but you could better assess the probability by looking at sales records for similar houses.

Abuses

One common abuse of probability is thinking that probabilities have "memories." For instance, if a coin is tossed eight times, the probability that it will land heads up all eight times is only about 0.004. However, if the coin has already been tossed seven times and has landed heads up each time, the probability that it will land heads up on the eighth time is 0.5. Each toss is independent of all other tosses. The coin does not "remember" that it has already landed heads up seven times.

Ethics

A human resources director for a company with 100 employees wants to show that her company is an equal opportunity employer of women and minorities. There are 40 women employees and 20 minority employees in the company. Nine of the women employees are minorities. Despite this fact, the director reports that 60% of the company is either a woman or a minority. If one employee is selected at random, the probability that the employee is a woman is 0.4 and the probability that the employee is a minority is 0.2. This does not mean, however, that the probability that a randomly selected employee is a woman or a minority is $0.4 + 0.2 = 0.6$, because nine employees belong to both groups. In this case, it would be ethically incorrect to omit this information from her report because these individuals would have been counted twice.

EXERCISES

1. ***Assuming That Probability Has a "Memory"*** A "Daily Number" lottery has a three-digit number from 000 to 999. You buy one ticket each day. Your number is 389.

- **a.** What is the probability of winning next Tuesday and Wednesday?
- **b.** You won on Tuesday. What's the probability of winning on Wednesday?
- **c.** You didn't win on Tuesday. What's the probability of winning on Wednesday?

2. ***Adding Probabilities Incorrectly*** A town has a population of 500 people. Suppose that the probability that a randomly chosen person owns a pickup truck is 0.25 and the probability that a randomly chosen person owns an SUV is 0.30. What can you say about the probability that a randomly chosen person owns a pickup or an SUV? Could this probability be 0.55? Could it be 0.60? Explain your reasoning.

3 CHAPTER SUMMARY

What did you **learn?**	EXAMPLE(S)	REVIEW EXERCISES
Section 3.1		
■ How to identify the sample space of a probability experiment and how to identify simple events	*1, 2*	*1–4*
■ How to use the Fundamental Counting Principle to find the number of ways two or more events can occur	*3, 4*	*5, 6*
■ How to distinguish among classical probability, empirical probability, and subjective probability	*5–8*	*7–12*
■ How to find the probability of the complement of an event and how to find other probabilities using the Fundamental Counting Principle	*9–11*	*13–16*
Section 3.2		
■ How to find conditional probabilities	*1*	*17, 18*
■ How to distinguish between independent and dependent events	*2*	*19–21*
■ How to use the Multiplication Rule to find the probability of two events occurring in sequence $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ if events are dependent $P(A \text{ and } B) = P(A) \cdot P(B)$ if events are independent	*3–5*	*22–24*
Section 3.3		
■ How to determine if two events are mutually exclusive	*1*	*25–27*
■ How to use the Addition Rule to find the probability of two events $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ if events are mutually exclusive	*2–5*	*28–40*
Section 3.4		
■ How to find the number of ways a group of objects can be arranged in order and the number of ways to choose several objects from a group without regard to order ${}_nP_r = \dfrac{n!}{(n-r)!}$ permutations of n objects taken r at a time $\dfrac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}$ distinguishable permutations ${}_nC_r = \dfrac{n!}{(n-r)!r!}$ combinations of n objects taken r at a time	*1–5*	*41–50*
■ How to use counting principles to find probabilities	*6–9*	*51–55*

3 REVIEW EXERCISES

SECTION 3.1

In Exercises 1–4, identify the sample space of the probability experiment and determine the number of outcomes in the event. Draw a tree diagram if it is appropriate.

1. *Experiment:* Tossing four coins
Event: Getting three heads

2. *Experiment:* Rolling 2 six-sided dice
Event: Getting a sum of 4 or 5

3. *Experiment:* Choosing a month of the year
Event: Choosing a month that begins with the letter J

4. *Experiment:* Guessing the gender(s) of the three children in a family
Event: The family has two boys

In Exercises 5 and 6, use the Fundamental Counting Principle.

5. A student must choose from 7 classes to take at 8:00 A.M., 4 classes to take at 9:00 A.M., and 3 classes to take at 10:00 A.M. How many ways can the student arrange the schedule?

6. The state of Virginia's license plates have three letters followed by four digits. Assuming that any letter or digit can be used, how many different license plates are possible?

In Exercises 7–12, classify the statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.

7. On the basis of prior counts, a quality control officer says there is a 0.05 probability that a randomly chosen part is defective.

8. The probability of randomly selecting five cards of the same suit from a standard deck is about 0.0005.

9. The chance that Corporation A's stock price will fall today is 75%.

10. The probability that a person can roll his or her tongue is 70%.

11. The probability of rolling 2 six-sided dice and getting a sum greater than 9 is $\frac{1}{6}$.

12. The chance that a randomly selected person in the United States is between 15 and 29 years old is about 21%. *(Source: U.S. Census Bureau)*

In Exercises 13 and 14, the table shows the approximate distribution of the sizes of firms for a recent year. Use the table to determine the probability of the event. (Adapted from U.S. Small Business Administration)

Number of employees	0 to 4	5 to 9	10 to 19	20 to 99	100 or more
Percent of firms	60.9%	17.6%	10.7%	9.0%	1.8%

13. What is the probability that a randomly selected firm will have at least 10 employees?

14. What is the probability that a randomly selected firm will have fewer than 20 employees?

Telephone Numbers *The telephone numbers for a region of a state have an area code of 570. The next seven digits represent the local telephone numbers for that region. A local telephone number cannot begin with a 0 or 1. Your cousin lives within the given area code.*

15. What is the probability of randomly generating your cousin's telephone number?

16. What is the probability of not randomly generating your cousin's telephone number?

SECTION 3.2

For Exercises 17 and 18, the two statements below summarize the results of a study on the use of plus/minus grading at North Carolina State University. It shows the percents of graduate and undergraduate students who received grades with pluses and minuses (for example, C+, A−, etc.). (Source: North Carolina State University)

- *Of all students who received one or more plus grades, 92% were undergraduates and 8% were graduates.*
- *Of all students who received one or more minus grades, 93% were undergraduates and 7% were graduates.*

17. Find the probability that a student is an undergraduate student, given that the student received a plus grade.

18. Find the probability that a student is a graduate student, given that the student received a minus grade.

In Exercises 19–21, decide whether the events are independent or dependent. Explain your reasoning.

19. Tossing a coin four times, getting four heads, and tossing it a fifth time and getting a head

20. Taking a driver's education course and passing the driver's license exam

21. Getting high grades and being awarded an academic scholarship

22. You are given that $P(A) = 0.35$ and $P(B) = 0.25$. Do you have enough information to find $P(A \text{ and } B)$? Explain.

In Exercises 23 and 24, find the probability of the sequence of events.

23. You are shopping, and your roommate has asked you to pick up toothpaste and dental rinse. However, your roommate did not tell you which brands to get. The store has eight brands of toothpaste and five brands of dental rinse. What is the probability that you will purchase the correct brands of both products? Is this an unusual event? Explain.

24. Your sock drawer has 18 folded pairs of socks, with 8 pairs of white, 6 pairs of black, and 4 pairs of blue. What is the probability, without looking in the drawer, that you will first select and remove a black pair, then select either a blue or a white pair? Is this an unusual event? Explain.

SECTION 3.3

In Exercises 25–27, decide if the events are mutually exclusive. Explain your reasoning.

25. Event A: Randomly select a red jelly bean from a jar.
Event B: Randomly select a yellow jelly bean from the same jar.

26. Event A: Randomly select a person who loves cats.
Event B: Randomly select a person who owns a dog.

27. Event A: Randomly select a U.S. adult registered to vote in Illinois.
Event B: Randomly select a U.S. adult registered to vote in Florida.

28. You are given that $P(A) = 0.15$ and $P(B) = 0.40$. Do you have enough information to find $P(A \text{ or } B)$? Explain.

29. A random sample of 250 working adults found that 37% access the Internet at work, 44% access the Internet at home, and 21% access the Internet at both work and home. What is the probability that a person in this sample selected at random accesses the Internet at home or at work?

30. A sample of automobile dealerships found that 19% of automobiles sold are silver, 22% of automobiles sold are sport utility vehicles (SUVs), and 16% of automobiles sold are silver SUVs. What is the probability that a randomly chosen sold automobile from this sample is silver or an SUV?

In Exercises 31–34, determine the probability.

31. A card is randomly selected from a standard deck. Find the probability that the card is between 4 and 8, inclusive, or is a club.

32. A card is randomly selected from a standard deck. Find the probability that the card is red or a queen.

33. A 12-sided die, numbered 1 to 12, is rolled. Find the probability that the roll results in an odd number or a number less than 4.

34. An 8-sided die, numbered 1 to 8, is rolled. Find the probability that the roll results in an even number or a number greater than 6.

In Exercises 35 and 36, use the pie chart, which shows the percent distribution of the number of students in traditional U.S. elementary schools. (Source: U.S. National Center for Education Statistics)

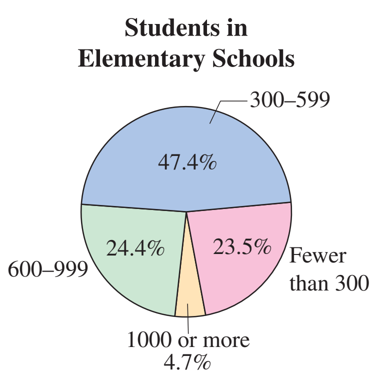
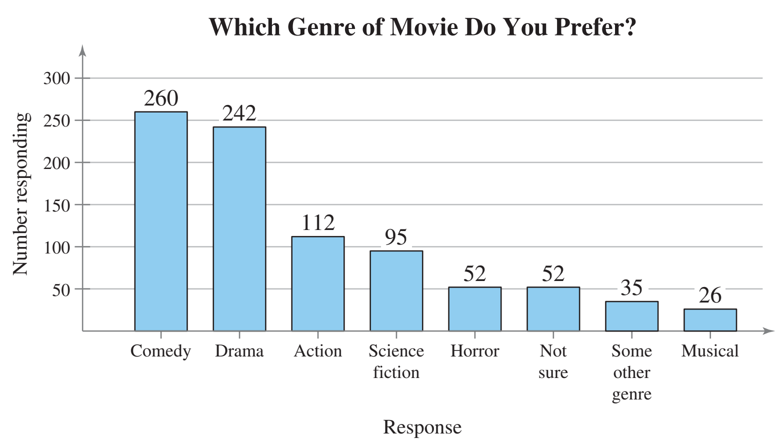


FIGURE FOR EXERCISES 35 AND 36

35. Find the probability of randomly selecting a school with 600 or more students.

36. Find the probability of randomly selecting a school with between 300 and 999 students, inclusive.

In Exercises 37–40, use the Pareto chart, which shows the results of a survey in which 874 adults were asked which genre of movie they preferred. (Adapted from Rasmussen Reports)



37. Find the probability of randomly selecting an adult from the sample who prefers an action movie or a horror movie.

38. Find the probability of randomly selecting an adult from the sample who prefers a drama or a musical.

39. Find the probability of randomly selecting an adult from the sample who does not prefer a comedy.

40. Find the probability of randomly selecting an adult from the sample who does not prefer a science fiction movie or an action movie.

SECTION 3.4

In Exercises 41–44, perform the indicated calculation.

41. ${}_{11}P_2$ **42.** ${}_{8}P_6$ **43.** ${}_{7}C_4$ **44.** $\dfrac{{}_{5}C_3}{{}_{10}C_3}$

45. Use a technology tool to find ${}_{50}P_5$.

46. Use a technology tool to find ${}_{38}C_{25}$.

In Exercises 47–50, use combinations and permutations.

47. Fifteen cyclists enter a race. In how many ways can they finish first, second, and third?

48. Five players on a basketball team must each choose a player on the opposing team to defend. In how many ways can they choose their defensive assignments?

49. A literary magazine editor must choose 4 short stories for this month's issue from 17 submissions. In how many ways can the editor choose this month's stories?

50. An employer must hire 2 people from a list of 13 applicants. In how many ways can the employer choose to hire the 2 people?

In Exercises 51–55, use counting principles to find the probability. Then tell whether the event can be considered unusual.

51. A full house consists of a three of one kind and two of another kind. Find the probability of a full house consisting of three kings and two queens.

52. A security code consists of three letters followed by one digit. The first letter cannot be an A, B, or C. What is the probability of guessing the security code in one trial?

53. A batch of 200 calculators contains 3 defective units. What is the probability that a sample of three calculators will have

(a) no defective calculators?

(b) all defective calculators?

(c) at least one defective calculator?

(d) at least one nondefective calculator?

54. A batch of 350 raffle tickets contains four winning tickets. You buy four tickets. What is the probability that you have

(a) no winning tickets?

(b) all of the winning tickets?

(c) at least one winning ticket?

(d) at least one nonwinning ticket?

55. A corporation has six male senior executives and four female senior executives. Four senior executives are chosen at random to attend a technology seminar. What is the probability of choosing

(a) four men?

(b) four women?

(c) two men and two women?

(d) one man and three women?

3 CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

1. The table shows the number (in thousands) of earned degrees, by level and gender, conferred in the United States in a recent year. *(Source: U.S. National Center for Education Statistics)*

		Gender		
		Male	**Female**	**Total**
Level of Degree	**Associate's**	275	453	728
	Bachelor's	650	875	1525
	Master's	238	366	604
	Doctoral	30	30	60
	Total	1193	1724	2917

A person who earned a degree in the year is randomly selected. Find the probability of selecting someone who

(a) earned a bachelor's degree.

(b) earned a bachelor's degree given that the person is a female.

(c) earned a bachelor's degree given that the person is not a female.

(d) earned an associate's degree or a bachelor's degree.

(e) earned a doctorate given that the person is a male.

(f) earned a master's degree or is a female.

(g) earned an associate's degree and is a male.

(h) is a female given that the person earned a bachelor's degree.

2. Which event(s) in Exercise 1 can be considered unusual? Explain your reasoning.

3. Decide if the events are mutually exclusive. Then decide if the events are independent or dependent. Explain your reasoning.

Event A: A golfer scoring the best round in a four-round tournament

Event B: Losing the golf tournament

4. A shipment of 250 netbooks contains 3 defective units. Determine how many ways a vending company can buy three of these units and receive

(a) no defective units.

(b) all defective units.

(c) at least one good unit.

5. In Exercise 4, find the probability of the vending company receiving

(a) no defective units.

(b) all defective units.

(c) at least one good unit.

6. The access code for a warehouse's security system consists of six digits. The first digit cannot be 0 and the last digit must be even. How many different codes are available?

7. From a pool of 30 candidates, the offices of president, vice president, secretary, and treasurer will be filled. In how many different ways can the offices be filled?

PUTTING IT ALL TOGETHER

Real Statistics — Real Decisions

You work for the company that runs the Powerball® lottery. Powerball is a lottery game in which five white balls are chosen from a drum containing 59 balls and one red ball is chosen from a drum containing 39 balls. To win the jackpot, a player must match all five white balls and the red ball. Other winners and their prizes are also shown in the table.

Working in the public relations department, you handle many inquiries from the media and from lottery players. You receive the following e-mail.

You list the probability of matching only the red ball as 1/62. I know from my statistics class that the probability of winning is the ratio of the number of successful outcomes to the total number of outcomes. Could you please explain why the probability of matching only the red ball is 1/62?

Your job is to answer this question, using the probability techniques you have learned in this chapter to justify your answer. In answering the question, assume only one ticket is purchased.

EXERCISES

1. *How Would You Do It?*

(a) How would you investigate the question about the probability of matching only the red ball?

(b) What statistical methods taught in this chapter would you use?

2. *Answering the Question*

Write an explanation that answers the question about the probability of matching only the red ball. Include in your explanation any probability formulas that justify your explanation.

3. *Another Question*

You receive another question asking how the overall probability of winning a prize in the Powerball lottery is determined. The overall probability of winning a prize in the Powerball lottery is 1/35. Write an explanation that answers the question and include any probability formulas that justify your explanation.

www.musl.com

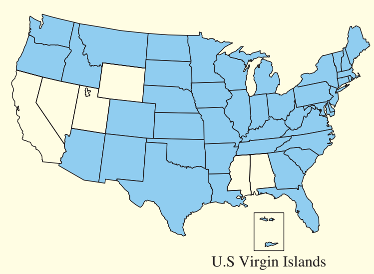
Powerball Winners and Prizes

Match	Prize	Approximate probability
5 white, 1 red	Jackpot	1/195,249,054
5 white	$200,000	1/5,138,133
4 white, 1 red	$10,000	1/723,145
4 white	$100	1/19,030
3 white, 1 red	$100	1/13,644
3 white	$7	1/359
2 white, 1 red	$7	1/787
1 white, 1 red	$4	1/123
1 red	$3	1/62

(Source: Multi-State Lottery Association)

Where Is Powerball Played?

Powerball is played in 42 states, Washington, D.C., and the U.S. Virgin Islands



(Source: Multi-State Lottery Association)

TECHNOLOGY

MINITAB | EXCEL | TI-83/84 PLUS

SIMULATION: COMPOSING MOZART VARIATIONS WITH DICE

Wolfgang Mozart (1756–1791) composed a wide variety of musical pieces. In his Musical Dice Game, he wrote a Wiener minuet with an almost endless number of variations. Each minuet has 16 bars. In the eighth and sixteenth bars, the player has a choice of two musical phrases. In each of the other 14 bars, the player has a choice of 11 phrases.

To create a minuet, Mozart suggested that the player toss 2 six-sided dice 16 times. For the eighth and sixteenth bars, choose Option 1 if the dice total is odd and Option 2 if it is even. For each of the other 14 bars, subtract 1 from the dice total. The following minuet is the result of the following sequence of numbers.

5	7	1	6	4	10	5	1
6	6	2	4	6	8	8	2

EXERCISES

1. How many phrases did Mozart write to create the Musical Dice Game minuet? Explain.
2. How many possible variations are there in Mozart's Musical Dice Game minuet? Explain.
3. Use technology to randomly select a number from 1 to 11.
   (a) What is the theoretical probability of each number from 1 to 11 occurring?
   (b) Use this procedure to select 100 integers from 1 to 11. Tally your results and compare them with the probabilities in part (a).
4. What is the probability of randomly selecting option 6, 7, or 8 for the first bar? For all 14 bars? Find each probability using (a) theoretical probability and (b) the results of Exercise 3(b).
5. Use technology to randomly select two numbers from 1, 2, 3, 4, 5, and 6. Find the sum and subtract 1 to obtain a total.
   (a) What is the theoretical probability of each total from 1 to 11?
   (b) Use this procedure to select 100 totals from 1 to 11. Tally your results and compare them with the probabilities in part (a).
6. Repeat Exercise 4 using the results of Exercise 5(b).

Extended solutions are given in the *Technology Supplement.*
Technical instruction is provided for MINITAB, Excel, and the TI-83/84 Plus.