4 CHAPTER

DISCRETE PROBABILITY DISTRIBUTIONS

4.1 Probability Distributions

4.2 Binomial Distributions

- ACTIVITY
- CASE STUDY

4.3 More Discrete Probability Distributions

- USES AND ABUSES
- REAL STATISTICS–REAL DECISIONS
- TECHNOLOGY

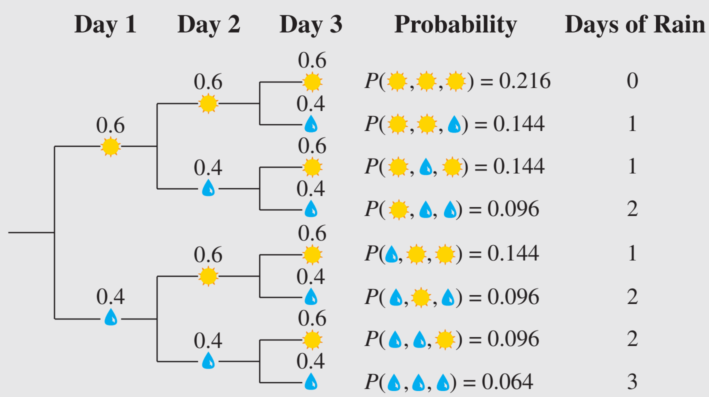
The National Climatic Data Center (NCDC) is the world's largest active archive of weather data. NCDC archives weather data from the Coast Guard, Federal Aviation Administration, Military Services, the National Weather Service, and voluntary observers.

WHERE YOU'VE BEEN

In Chapters 1 through 3, you learned how to collect and describe data and how to find the probability of an event. These skills are used in many different types of careers. For instance, data about climatic conditions are used to analyze and forecast the weather throughout the world. On a typical day, aircraft, National Weather Service cooperative observers, radar, remote sensing systems, satellites, ships, weather balloons, wind profilers, and a variety of other data-collection devices work together to provide meteorologists with data that are used to forecast the weather. Even with this much data, meteorologists cannot forecast the weather with certainty. Instead, they assign probabilities to certain weather conditions. For instance, a meteorologist might determine that there is a 40% chance of rain (based on the relative frequency of rain under similar weather conditions).

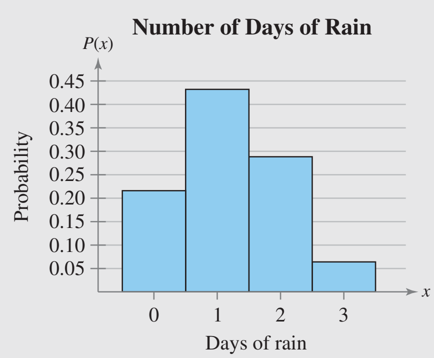
WHERE YOU'RE GOING

In Chapter 4, you will learn how to create and use probability distributions. Knowing the shape, center, and variability of a probability distribution will enable you to make decisions in inferential statistics. You are a meteorologist working on a three-day forecast. Assuming that having rain on one day is independent of having rain on another day, you have determined that there is a 40% probability of rain (and a 60% probability of no rain) on each of the three days. What is the probability that it will rain on 0, 1, 2, or 3 of the days? To answer this, you can create a probability distribution for the possible outcomes.



Using the *Addition Rule* with the probabilities in the tree diagram, you can determine the probabilities of having rain on various numbers of days. You can then use this information to graph a probability distribution.

Probability Distribution		
Days of rain	Tally	Probability
0	1	0.216
1	3	0.432
2	3	0.288
3	1	0.064



4.1 Probability Distributions

WHAT YOU SHOULD LEARN

- How to distinguish between discrete random variables and continuous random variables
- How to construct a discrete probability distribution and its graph
- How to determine if a distribution is a probability distribution
- How to find the mean, variance, and standard deviation of a discrete probability distribution
- How to find the expected value of a discrete probability distribution

Random Variables ▸ Discrete Probability Distributions ▸ Mean, Variance, and Standard Deviation ▸ Expected Value

RANDOM VARIABLES

The outcome of a probability experiment is often a count or a measure. When this occurs, the outcome is called a *random variable*.

DEFINITION

A **random variable** x represents a numerical value associated with each outcome of a probability experiment.

The word *random* indicates that x is determined by chance. There are two types of random variables: *discrete* and *continuous*.

DEFINITION

A random variable is **discrete** if it has a finite or countable number of possible outcomes that can be listed.

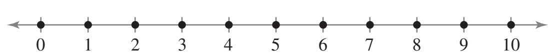
A random variable is **continuous** if it has an uncountable number of possible outcomes, represented by an interval on the number line.

You conduct a study of the number of calls a telemarketer makes in one day. The possible values of the random variable x are 0, 1, 2, 3, 4, and so on. Because the set of possible outcomes

$$\{0, 1, 2, 3, \ldots\}$$

can be listed, x is a discrete random variable. You can represent its values as points on a number line.

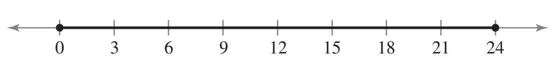
Number of Calls (Discrete)



x can have only whole number values: 0, 1, 2, 3, ...

A different way to conduct the study would be to measure the time (in hours) a telemarketer spends making calls in one day. Because the time spent making calls can be any number from 0 to 24 (including fractions and decimals), x is a continuous random variable. You can represent its values with an interval on a number line.

Hours Spent on Calls (Continuous)



x can have any value between 0 and 24.

When a random variable is discrete, you can list the possible values it can assume. However, it is impossible to list all values for a continuous random variable.

EXAMPLE 1

Discrete Variables and Continuous Variables

Decide whether the random variable x is discrete or continuous. Explain your reasoning.

1. Let x represent the number of Fortune 500 companies that lost money in the previous year.
2. Let x represent the volume of gasoline in a 21-gallon tank.

Solution

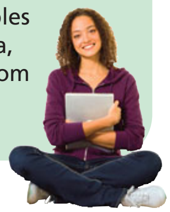
1. The number of companies that lost money in the previous year can be counted.

   $$\{0, 1, 2, 3, \ldots, 500\}$$

   So, x is a *discrete* random variable.
2. The amount of gasoline in the tank can be any volume between 0 gallons and 21 gallons. So, x is a *continuous* random variable.

STUDY TIP

In most practical applications, discrete random variables represent counted data, while continuous random variables represent measured data.



Try It Yourself 1

Decide whether the random variable x is discrete or continuous. Explain your reasoning.

1. Let x represent the speed of a Space Shuttle.
2. Let x represent the number of calves born on a farm in one year.

a. Decide if x represents *counted* data or *measured* data.
b. Make a *conclusion* and *explain* your reasoning.

Answer: Page A36

INSIGHT

Values of variables such as age, height, and weight are usually rounded to the nearest year, inch, or pound. However, these values represent measured data, so they are continuous random variables.

It is important that you can distinguish between discrete and continuous random variables because different statistical techniques are used to analyze each. The remainder of this chapter focuses on discrete random variables and their probability distributions. You will study continuous distributions later.

DISCRETE PROBABILITY DISTRIBUTIONS

Each value of a discrete random variable can be assigned a probability. By listing each value of the random variable with its corresponding probability, you are forming a *discrete probability distribution*.

DEFINITION

A **discrete probability distribution** lists each possible value the random variable can assume, together with its probability. A discrete probability distribution must satisfy the following conditions.

IN WORDS	IN SYMBOLS
1. The probability of each value of the discrete random variable is between 0 and 1, inclusive.	$0 \leq P(x) \leq 1$
2. The sum of all the probabilities is 1.	$\Sigma P(x) = 1$

Because probabilities represent relative frequencies, a discrete probability distribution can be graphed with a relative frequency histogram.

GUIDELINES

Constructing a Discrete Probability Distribution

Let x be a discrete random variable with possible outcomes $x_1, x_2, \ldots, x_n$.

1. Make a frequency distribution for the possible outcomes.
2. Find the sum of the frequencies.
3. Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
4. Check that each probability is between 0 and 1, inclusive, and that the sum of all the probabilities is 1.

EXAMPLE 2

SC Report 16

Constructing and Graphing a Discrete Probability Distribution

An industrial psychologist administered a personality inventory test for passive-aggressive traits to 150 employees. Each individual was given a score from 1 to 5, where 1 was extremely passive and 5 extremely aggressive. A score of 3 indicated neither trait. The results are shown at the left. Construct a probability distribution for the random variable x. Then graph the distribution using a histogram.

Frequency Distribution

Score, x	Frequency, f
1	24
2	33
3	42
4	30
5	21

Solution

Divide the frequency of each score by the total number of individuals in the study to find the probability for each value of the random variable.

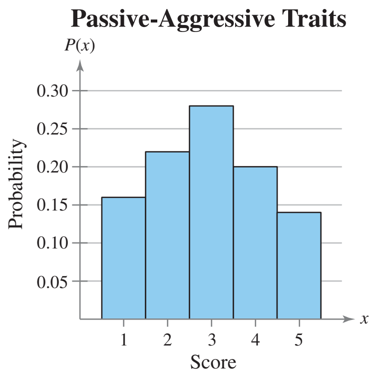
$$P(1) = \frac{24}{150} = 0.16 \qquad P(2) = \frac{33}{150} = 0.22 \qquad P(3) = \frac{42}{150} = 0.28$$

$$P(4) = \frac{30}{150} = 0.20 \qquad P(5) = \frac{21}{150} = 0.14$$

The discrete probability distribution is shown in the following table.

x	1	2	3	4	5
$P(x)$	0.16	0.22	0.28	0.20	0.14

Note that $0 \le P(x) \le 1$ and $\sum P(x) = 1$.



The histogram is shown at the left. Because the width of each bar is one, the area of each bar is equal to the probability of a particular outcome. Also, the probability of an event corresponds to the sum of the areas of the outcomes included in the event. For instance, the probability of the event "having a score of 2 or 3" is equal to the sum of the areas of the second and third bars,

$$(1)(0.22) + (1)(0.28) = 0.22 + 0.28 = 0.50.$$

Interpretation You can see that the distribution is approximately symmetric.

Try It Yourself 2

A company tracks the number of sales new employees make each day during a 100-day probationary period. The results for one new employee are shown at the left. Construct and graph a probability distribution.

Frequency Distribution

Sales per day, x	Number of days, f
0	16
1	19
2	15
3	21
4	9
5	10
6	8
7	2

a. Find the *probability* of each outcome.
b. Organize the probabilities in a *probability distribution*.
c. Graph the probability distribution using a *histogram*.

Answer: Page A36

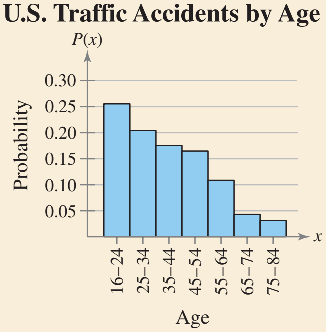
Probability Distribution

Days of rain, x	Probability, $P(x)$
0	0.216
1	0.432
2	0.288
3	0.064

PICTURING THE WORLD

In a recent year in the United States, nearly 11 million traffic accidents were reported to the police. A histogram of traffic accidents for various age groups from 16 to 84 is shown. (Adapted from National Safety Council)

U.S. Traffic Accidents by Age



Estimate the probability that a randomly selected person involved in a traffic accident is in the 16 to 34 age group.

EXAMPLE 3

Verifying Probability Distributions

Verify that the distribution at the left (see page 189) is a probability distribution.

Solution

If the distribution is a probability distribution, then (1) each probability is between 0 and 1, inclusive, and (2) the sum of the probabilities equals 1.

1. Each probability is between 0 and 1.

2. $\Sigma P(x) = 0.216 + 0.432 + 0.288 + 0.064$

$= 1.$

Interpretation Because both conditions are met, the distribution is a probability distribution.

Try It Yourself 3

Verify that the distribution you constructed in Try It Yourself 2 is a probability distribution.

a. Verify that the *probability* of each outcome is between 0 and 1, inclusive.
b. Verify that the *sum* of all the probabilities is 1.
c. Make a *conclusion*.

Answer: Page A36

EXAMPLE 4

Identifying Probability Distributions

Decide whether the distribution is a probability distribution. Explain your reasoning.

1.

x	5	6	7	8
$P(x)$	0.28	0.21	0.43	0.15

2.

x	1	2	3	4
$P(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{4}$	-1

Solution

1. Each probability is between 0 and 1, but the sum of all the probabilities is 1.07, which is greater than 1. So, it is *not* a probability distribution.

2. The sum of all the probabilities is equal to 1, but $P(3)$ and $P(4)$ are not between 0 and 1. So, it is *not* a probability distribution. Probabilities can never be negative or greater than 1.

Try It Yourself 4

Decide whether the distribution is a probability distribution. Explain your reasoning.

1.

x	5	6	7	8
$P(x)$	$\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

2.

x	1	2	3	4
$P(x)$	0.09	0.36	0.49	0.06

a. Verify that the *probability* of each outcome is between 0 and 1.
b. Verify that the *sum* of all the probabilities is 1.
c. Make a *conclusion*.

Answer: Page A36

MEAN, VARIANCE, AND STANDARD DEVIATION

You can measure the center of a probability distribution with its mean and measure the variability with its variance and standard deviation. The mean of a discrete random variable is defined as follows.

MEAN OF A DISCRETE RANDOM VARIABLE

The **mean** of a discrete random variable is given by

$$\mu = \Sigma x P(x).$$

Each value of x is multiplied by its corresponding probability and the products are added.

The mean of a random variable represents the "theoretical average" of a probability experiment and sometimes is not a possible outcome. If the experiment were performed many thousands of times, the mean of all the outcomes would be close to the mean of the random variable.

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

EXAMPLE 5

Finding the Mean of a Probability Distribution

The probability distribution for the personality inventory test for passive-aggressive traits discussed in Example 2 is given at the left. Find the mean score. What can you conclude?

Solution

Use a table to organize your work, as shown below. From the table, you can see that the mean score is approximately 2.9. A score of 3 represents an individual who exhibits neither passive nor aggressive traits. The mean is slightly under 3.

x	$P(x)$	$xP(x)$
1	0.16	$1(0.16) = 0.16$
2	0.22	$2(0.22) = 0.44$
3	0.28	$3(0.28) = 0.84$
4	0.20	$4(0.20) = 0.80$
5	0.14	$5(0.14) = 0.70$
	$\Sigma P(x) = 1$	$\Sigma x P(x) = 2.94$ ← Mean

Interpretation You can conclude that the mean personality trait is neither extremely passive nor extremely aggressive, but is slightly closer to passive.

Try It Yourself 5

Find the mean of the probability distribution you constructed in Try It Yourself 2. What can you conclude?

a. Find the *product* of each random outcome and its corresponding probability.
b. Find the *sum* of the products.
c. Make a *conclusion*.

Answer: Page A36

STUDY TIP

Notice that the mean in Example 5 is rounded to one decimal place. This rounding was done because the mean of a probability distribution should be rounded to one more decimal place than was used for the random variable x. This *round-off rule* is also used for the variance and standard deviation of a probability distribution.

Although the mean of the random variable of a probability distribution describes a typical outcome, it gives no information about how the outcomes vary. To study the variation of the outcomes, you can use the variance and standard deviation of the random variable of a probability distribution.

STUDY TIP

A shortcut formula for the variance of a probability distribution is

$$\sigma^2 = [\Sigma x^2 P(x)] - \mu^2.$$

VARIANCE AND STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

The **variance** of a discrete random variable is

$$\sigma^2 = \Sigma(x - \mu)^2 P(x).$$

The **standard deviation** is

$$\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x - \mu)^2 P(x)}.$$

EXAMPLE 6

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

Finding the Variance and Standard Deviation

The probability distribution for the personality inventory test for passive-aggressive traits discussed in Example 2 is given at the left. Find the variance and standard deviation of the probability distribution.

Solution

From Example 5, you know that before rounding, the mean of the distribution is $\mu = 2.94$. Use a table to organize your work, as shown below.

x	$P(x)$	$x - \mu$	$(x - \mu)^2$	$P(x)(x - \mu)^2$
1	0.16	−1.94	3.764	0.602
2	0.22	−0.94	0.884	0.194
3	0.28	0.06	0.004	0.001
4	0.20	1.06	1.124	0.225
5	0.14	2.06	4.244	0.594
	$\Sigma P(x) = 1$			$\Sigma P(x)(x - \mu)^2 = 1.616$

Variance

So, the variance is

$$\sigma^2 = 1.616 \approx 1.6$$

and the standard deviation is

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.616} \approx 1.3.$$

Interpretation Most of the data values differ from the mean by no more than 1.3.

Try It Yourself 6

Find the variance and standard deviation of the probability distribution constructed in Try It Yourself 2.

a. For each value of x, find the *square of the deviation* from the mean and multiply that value by the corresponding probability of x.
b. Find the *sum of the products* found in part (a) for the variance.
c. Take the *square root of the variance* to find the standard deviation.
d. *Interpret* the results.

Answer: Page A36

STUDY TIP

Detailed instructions for using MINITAB, Excel, and the TI-83/84 Plus are shown in the Technology Guide that accompanies this text. Here are instructions for finding the mean and standard deviation of a discrete random variable of a probability distribution on a TI-83/84 Plus.

STAT

Choose the EDIT menu.

1: Edit

Enter the possible values of the discrete random variable x in L1. Enter the probabilities $P(x)$ in L2.

STAT

Choose the CALC menu.

1: 1–Var Stats

ENTER

2nd L1 , 2nd L2

ENTER

EXPECTED VALUE

The mean of a random variable represents what you would expect to happen over thousands of trials. It is also called the *expected value*.

DEFINITION

The **expected value** of a discrete random variable is equal to the mean of the random variable.

$$\text{Expected Value} = E(x) = \mu = \sum xP(x)$$

Although probabilities can never be negative, the expected value of a random variable can be negative.

INSIGHT

In most applications, an expected value of 0 has a practical interpretation. For instance, in games of chance, an expected value of 0 implies that a game is fair (an unlikely occurrence!). In a profit and loss analysis, an expected value of 0 represents the break-even point.

EXAMPLE 7

Finding an Expected Value

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. What is the expected value of your gain?

Solution

To find the gain for each prize, subtract the price of the ticket from the prize. For instance, your gain for the $500 prize is

$$\$500 - \$2 = \$498$$

and your gain for the $250 prize is

$$\$250 - \$2 = \$248.$$

Write a probability distribution for the possible gains (or outcomes).

Gain, *x*	$498	$248	$148	$73	−$2
Probability, *P*(*x*)	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1496}{1500}$

Then, using the probability distribution, you can find the expected value.

$$\begin{aligned} E(x) &= \sum xP(x) \\ &= \$498 \cdot \frac{1}{1500} + \$248 \cdot \frac{1}{1500} + \$148 \cdot \frac{1}{1500} + \$73 \cdot \frac{1}{1500} + (-\$2) \cdot \frac{1496}{1500} \\ &= -\$1.35 \end{aligned}$$

Interpretation Because the expected value is negative, you can expect to lose an average of $1.35 for each ticket you buy.

Try It Yourself 7

At a raffle, 2000 tickets are sold at $5 each for five prizes of $2000, $1000, $500, $250, and $100. You buy one ticket. What is the expected value of your gain?

a. Find the *gain* for each prize.
b. Write a *probability distribution* for the possible gains.
c. Find the *expected value*.
d. *Interpret* the results.

Answer: Page A36

4.1 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. What is a random variable? Give an example of a discrete random variable and a continuous random variable. Justify your answer.

2. What is a discrete probability distribution? What are the two conditions that determine a probability distribution?

3. Is the expected value of the probability distribution of a random variable always one of the possible values of x? Explain.

4. What is the significance of the mean of a probability distribution?

True or False? *In Exercises 5–8, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

5. In most applications, continuous random variables represent counted data, while discrete random variables represent measured data.

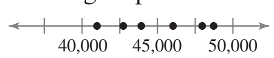
6. For a random variable x, the word *random* indicates that the value of x is determined by chance.

7. The mean of a random variable represents the "theoretical average" of a probability experiment and sometimes is not a possible outcome.

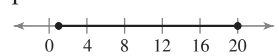
8. The expected value of a discrete random variable is equal to the standard deviation of the random variable.

Graphical Analysis *In Exercises 9–12, decide whether the graph represents a discrete random variable or a continuous random variable. Explain your reasoning.*

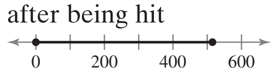
9. The attendance at concerts for a rock group



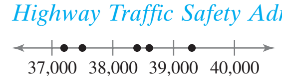
10. The length of time student-athletes practice each week



11. The distance a baseball travels after being hit



12. The annual traffic fatalities in the United States *(Source: U.S. National Highway Traffic Safety Administration)*



Distinguishing Between Discrete and Continuous Random Variables *In Exercises 13–20, decide whether the random variable x is discrete or continuous. Explain your reasoning.*

13. Let x represent the number of books in a university library.

14. Let x represent the length of time it takes to get to work.

15. Let x represent the volume of blood drawn for a blood test.

16. Let x represent the number of tornadoes in the month of June in Oklahoma.

17. Let x represent the number of messages posted each month on a social networking website.

18. Let x represent the tension at which a randomly selected guitar's strings have been strung.

19. Let x represent the amount of snow (in inches) that fell in Nome, Alaska last winter.

20. Let x represent the total number of die rolls required for an individual to roll a five.

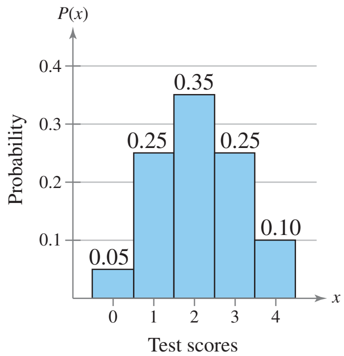


FIGURE FOR EXERCISE 21

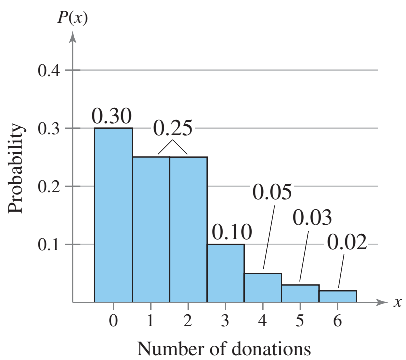


FIGURE FOR EXERCISE 22

USING AND INTERPRETING CONCEPTS

21. Employee Testing A company gave psychological tests to prospective employees. The random variable x represents the possible test scores. Use the histogram to find the probability that a person selected at random from the survey's sample had a test score of (a) more than two and (b) less than four.

22. Blood Donations A survey asked a sample of people how many times they donate blood each year. The random variable x represents the number of donations in one year. Use the histogram to find the probability that a person selected at random from the survey's sample donated blood (a) more than once in a year and (b) less than three times in a year.

Determining a Missing Probability *In Exercises 23 and 24, determine the probability distribution's missing probability value.*

23.

x	0	1	2	3	4
$P(x)$	0.07	0.20	0.38	?	0.13

24.

x	0	1	2	3	4	5	6
$P(x)$	0.05	?	0.23	0.21	0.17	0.11	0.08

Identifying Probability Distributions *In Exercises 25 and 26, decide whether the distribution is a probability distribution. If it is not a probability distribution, identify the property (or properties) that are not satisfied.*

25. Tires A mechanic checked the tire pressures on each car that he worked on for one week. The random variable x represents the number of tires that were underinflated.

x	0	1	2	3	4
$P(x)$	0.30	0.25	0.25	0.15	0.05

26. Quality Control A quality inspector checked for imperfections in rolls of fabric for one week. The random variable x represents the number of imperfections found.

x	0	1	2	3	4	5
$P(x)$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{50}$	$\frac{1}{100}$

Constructing Probability Distributions *In Exercises 27–32, (a) use the frequency distribution to construct a probability distribution, (b) graph the probability distribution using a histogram and describe its shape, (c) find the mean, variance, and standard deviation of the probability distribution, and (d) interpret the results in the context of the real-life situation.*

27. Dogs The number of dogs per household in a small town

Dogs	0	1	2	3	4	5
Households	1491	425	168	48	29	14

28. Baseball The number of games played in the World Series from 1903 to 2009 *(Source: Major League Baseball)*

Games played	4	5	6	7	8
Frequency	20	23	23	36	3

29. Televisions The number of televisions per household in a small town

Televisions	0	1	2	3
Households	26	442	728	1404

30. Camping Chairs The number of defects per batch of camping chairs inspected

Defects	0	1	2	3	4	5
Batches	95	113	87	64	13	8

31. Overtime Hours The number of overtime hours worked in one week per employee

Overtime hours	0	1	2	3	4	5	6
Employees	6	12	29	57	42	30	16

32. Extracurricular Activities The number of school-related extracurricular activities per student

Activities	0	1	2	3	4	5	6	7
Students	19	39	52	57	68	41	27	17

33. Writing The expected value of an accountant's profit and loss analysis is 0. Explain what this means.

34. Writing In a game of chance, what is the relationship between a "fair bet" and its expected value? Explain.

Finding Expected Value

In Exercises 35–40, use the probability distribution or histogram to find the (a) mean, (b) variance, (c) standard deviation, and (d) expected value of the probability distribution, and (e) interpret the results.

35. Quiz Students in a class take a quiz with eight questions. The random variable x represents the number of questions answered correctly.

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.02	0.02	0.06	0.06	0.08	0.22	0.30	0.16	0.08

36. 911 Calls A 911 service center recorded the number of calls received per hour. The random variable x represents the number of calls per hour for one week.

x	0	1	2	3	4	5	6	7
$P(x)$	0.01	0.10	0.26	0.33	0.18	0.06	0.03	0.03

37. Hurricanes The histogram shows the distribution of hurricanes that have hit the U.S. mainland by category, with 1 the weakest level and 5 the strongest. *(Source: Weather Research Center)*

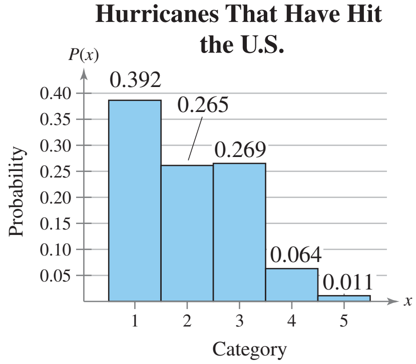


FIGURE FOR EXERCISE 37

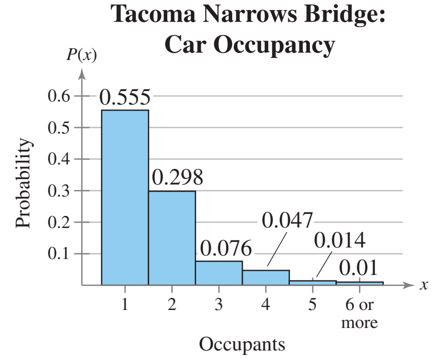


FIGURE FOR EXERCISE 38

38. Car Occupancy The histogram shows the distribution of occupants in cars crossing the Tacoma Narrows Bridge in Washington each week. *(Adapted from Washington State Department of Transportation)*

39. Household Size The histogram shows the distribution of household sizes in the United States for a recent year. *(Adapted from U.S. Census Bureau)*

40. Carpooling The histogram shows the distribution of carpooling by the number of cars per household. *(Adapted from Federal Highway Administration)*

41. Finding Probabilities Use the probability distribution you made for Exercise 27 to find the probability of randomly selecting a household that has (a) fewer than two dogs, (b) at least one dog, and (c) between one and three dogs, inclusive.

42. Finding Probabilities Use the probability distribution you made for Exercise 28 to find the probability of randomly selecting a World Series that consisted of (a) four games, (b) at least five games, and (c) between four and six games, inclusive.

43. Unusual Values A person lives in a household with three dogs and claims that having three dogs is not unusual. Use the information in Exercise 27 to determine if this person is correct. Explain your reasoning.

44. Unusual Values A person randomly chooses a World Series in which eight games were played and claims that this is an unusual event. Use the information in Exercise 28 to determine if this person is correct. Explain you reasoning.

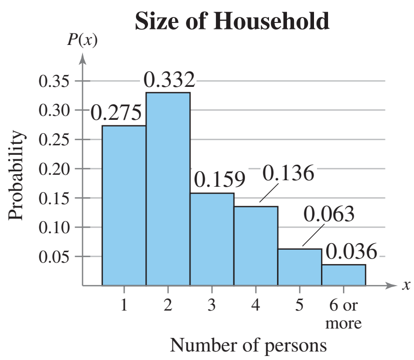


FIGURE FOR EXERCISE 39

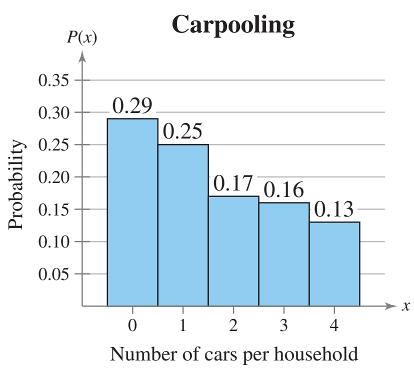


FIGURE FOR EXERCISE 40

Games of Chance *In Exercises 45 and 46, find the expected net gain to the player for one play of the game. If x is the net gain to a player in a game of chance, then $E(x)$ is usually negative. This value gives the average amount per game the player can expect to lose.*

45. In American roulette, the wheel has the 38 numbers

$$00, 0, 1, 2, \ldots, 34, 35, \text{and } 36$$

marked on equally spaced slots. If a player bets $1 on a number and wins, then the player keeps the dollar and receives an additional 35 dollars. Otherwise, the dollar is lost.

46. A charity organization is selling $5 raffle tickets as part of a fund-raising program. The first prize is a trip to Mexico valued at $3450, and the second prize is a weekend spa package valued at $750. The remaining 20 prizes are $25 gas cards. The number of tickets sold is 6000.

SC *In Exercises 47 and 48, use StatCrunch to (a) construct and graph a probability distribution and (b) describe its shape.*

47. Computers The number of computers per household in a small town

Computers	0	1	2	3
Households	300	280	95	20

48. Students The enrollments (in thousands) for grades 1 through 8 in the United States for a recent year *(Source: U.S. National Center for Education Statistics)*

Grade	1	2	3	4	5	6	7	8
Enrollment	3750	3640	3627	3585	3601	3660	3715	3765

EXTENDING CONCEPTS

Linear Transformation of a Random Variable *In Exercises 49 and 50, use the following information. For a random variable x, a new random variable y can be created by applying a* ***linear transformation*** $y = a + bx$, *where a and b are constants. If the random variable x has mean* μ_x *and standard deviation* σ_x, *then the mean, variance, and standard deviation of y are given by the following formulas.*

$$\mu_y = a + b\mu_x \qquad \sigma_y{}^2 = b^2\sigma_x{}^2 \qquad \sigma_y = |b|\sigma_x$$

49. The mean annual salary of employees at a company is \$36,000. At the end of the year, each employee receives a \$1000 bonus and a 5% raise (based on salary). What is the new mean annual salary (including the bonus and raise) of the employees?

50. The mean annual salary of employees at a company is \$36,000 with a variance of 15,202,201. At the end of the year, each employee receives a \$2000 bonus and a 4% raise (based on salary). What is the standard deviation of the new salaries?

Independent and Dependent Random Variables *Two random variables x and y are* ***independent*** *if the value of x does not affect the value of y. If the variables are not independent, they are* ***dependent.*** *A new random variable can be formed by finding the sum or difference of random variables. If a random variable x has mean* μ_x *and a random variable y has mean* μ_y, *then the means of the sum and difference of the variables are given by the following equations.*

$$\mu_{x+y} = \mu_x + \mu_y \qquad \mu_{x-y} = \mu_x - \mu_y$$

If random variables are independent, then the variance and standard deviation of the sum or difference of the random variables can be found. So, if a random variable x has variance $\sigma^2{}_x$ *and a random variable y has variance* $\sigma^2{}_y$, *then the variances of the sum and difference of the variables are given by the following equations. Note that the variance of the difference is the sum of the variances.*

$$\sigma^2{}_{x+y} = \sigma^2{}_x + \sigma^2{}_y \qquad \sigma^2{}_{x-y} = \sigma^2{}_x + \sigma^2{}_y$$

In Exercises 51 and 52, the distribution of SAT scores for college-bound male seniors has a mean of 1524 and a standard deviation of 317. The distribution of SAT scores for college-bound female seniors has a mean of 1496 and a standard deviation of 307. One male and one female are randomly selected. Assume their scores are independent. (Source: The College Board)

51. What is the average sum of their scores? What is the average difference of their scores?

52. What is the standard deviation of the difference in their scores?

4.2 Binomial Distributions

WHAT YOU SHOULD LEARN

- How to determine if a probability experiment is a binomial experiment
- How to find binomial probabilities using the binomial probability formula
- How to find binomial probabilities using technology, formulas, and a binomial probability table
- How to graph a binomial distribution
- How to find the mean, variance, and standard deviation of a binomial probability distribution

Binomial Experiments ▸ Binomial Probability Formula ▸ Finding Binomial Probabilities ▸ Graphing Binomial Distributions ▸ Mean, Variance, and Standard Deviation

BINOMIAL EXPERIMENTS

There are many probability experiments for which the results of each trial can be reduced to two outcomes: success and failure. For instance, when a basketball player attempts a free throw, he or she either makes the basket or does not. Probability experiments such as these are called *binomial experiments*.

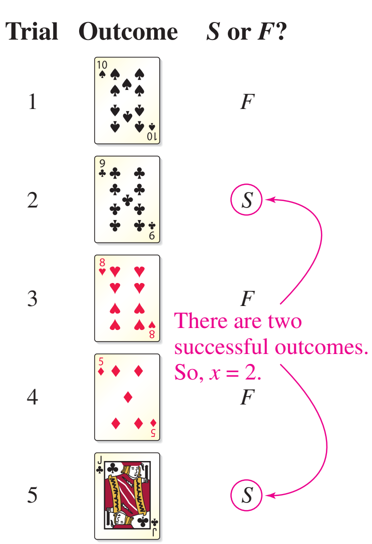
DEFINITION

A **binomial experiment** is a probability experiment that satisfies the following conditions.

1. The experiment is repeated for a fixed number of trials, where each trial is independent of the other trials.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success (S) or as a failure (F).
3. The probability of a success $P(S)$ is the same for each trial.
4. The random variable x counts the number of successful trials.

NOTATION FOR BINOMIAL EXPERIMENTS

SYMBOL	DESCRIPTION
n	The number of times a trial is repeated
$p = P(S)$	The probability of success in a single trial
$q = P(F)$	The probability of failure in a single trial $(q = 1 - p)$
x	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.



Here is a simple example of a binomial experiment. From a standard deck of cards, you pick a card, note whether it is a club or not, and replace the card. You repeat the experiment five times, so $n = 5$. The outcomes of each trial can be classified in two categories: S = selecting a club and F = selecting another suit. The probabilities of success and failure are

$$p = P(S) = \frac{1}{4} \quad \text{and} \quad q = P(F) = \frac{3}{4}.$$

The random variable x represents the number of clubs selected in the five trials. So, the possible values of the random variable are

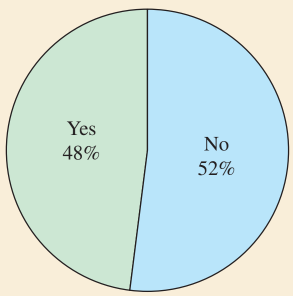
0, 1, 2, 3, 4, and 5.

For instance, if $x = 2$, then exactly two of the five cards are clubs and the other three are not clubs. An example of an experiment with $x = 2$ is shown at the left. Note that x is a discrete random variable because its possible values can be listed.

PICTURING THE WORLD

In a recent survey of U.S. adults who used the social networking website Twitter were asked if they had ever posted comments about their personal lives. The respondents' answers were either yes or no. (Adapted from Zogby International)

Survey question: Have you ever posted comments about your personal life on Twitter?



Why is this a binomial experiment? Identify the probability of success p. Identify the probability of failure q.

EXAMPLE 1

Identifying and Understanding Binomial Experiments

Decide whether the experiment is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not, explain why.

1. A certain surgical procedure has an 85% chance of success. A doctor performs the procedure on eight patients. The random variable represents the number of successful surgeries.
2. A jar contains five red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, *without replacement*. The random variable represents the number of red marbles.

Solution

1. The experiment is a binomial experiment because it satisfies the four conditions of a binomial experiment. In the experiment, each surgery represents one trial. There are eight surgeries, and each surgery is independent of the others. There are only two possible outcomes for each surgery—either the surgery is a success or it is a failure. Also, the probability of success for each surgery is 0.85. Finally, the random variable x represents the number of successful surgeries.

$$n = 8$$
$$p = 0.85$$
$$q = 1 - 0.85 = 0.15$$
$$x = 0, 1, 2, 3, 4, 5, 6, 7, 8$$

2. The experiment is not a binomial experiment because it does not satisfy all four conditions of a binomial experiment. In the experiment, each marble selection represents one trial, and selecting a red marble is a success. When the first marble is selected, the probability of success is 5/20. However, because the marble is not replaced, the probability of success for subsequent trials is no longer 5/20. So, the trials are not independent, and the probability of a success is not the same for each trial.

Try It Yourself 1

Decide whether the following is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not, explain why.

> You take a multiple-choice quiz that consists of 10 questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. The random variable represents the number of correct answers.

a. Identify a *trial* of the experiment and what is a success.
b. Decide if the experiment *satisfies the four conditions* of a binomial experiment.
c. Make a *conclusion* and *identify* n, p, q, and the possible values of x, if possible.

Answer: Page A36

INSIGHT

In the binomial probability formula, ${}_nC_x$ determines the number of ways of getting x successes in n trials, regardless of order.

$${}_nC_x = \frac{n!}{(n-x)!\,x!}$$

▸ BINOMIAL PROBABILITY FORMULA

There are several ways to find the probability of x successes in n trials of a binomial experiment. One way is to use a tree diagram and the Multiplication Rule. Another way is to use the binomial probability formula.

BINOMIAL PROBABILITY FORMULA

In a binomial experiment, the probability of exactly x successes in n trials is

$$P(x) = {}_nC_x\, p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}.$$

EXAMPLE 2

SC Report 17

▸ Finding Binomial Probabilities

Microfracture knee surgery has a 75% chance of success on patients with degenerative knees. The surgery is performed on three patients. Find the probability of the surgery being successful on exactly two patients. *(Source: Illinois Sportsmedicine and Orthopedic Center)*

STUDY TIP

Recall that $n!$ is read "n factorial" and represents the product of all integers from n to 1. For instance,

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$= 120.$$

▸ Solution

Method 1: Draw a tree diagram and use the Multiplication Rule.

1st Surgery	2nd Surgery	3rd Surgery	Outcome	Number of Successes	Probability
S	S	S	SSS	3	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{27}{64}$
S	S	F	SSF	2	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{9}{64}$
S	F	S	SFS	2	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{9}{64}$
S	F	F	SFF	1	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{3}{64}$
F	S	S	FSS	2	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{9}{64}$
F	S	F	FSF	1	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{3}{64}$
F	F	S	FFS	1	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{3}{64}$
F	F	F	FFF	0	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{64}$

There are three outcomes that have exactly two successes, and each has a probability of $\frac{9}{64}$. So, the probability of a successful surgery on exactly two patients is $3\left(\frac{9}{64}\right) \approx 0.422$.

Method 2: Use the binomial probability formula.

In this binomial experiment, the values of n, p, q, and x are $n = 3$, $p = \frac{3}{4}$, $q = \frac{1}{4}$, and $x = 2$. The probability of exactly two successful surgeries is

$$P(2 \text{ successful surgeries}) = \frac{3!}{(3-2)!2!}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1$$
$$= 3\left(\frac{9}{16}\right)\left(\frac{1}{4}\right) = 3\left(\frac{9}{64}\right) = \frac{27}{64} \approx 0.422.$$

▸ Try It Yourself 2

A card is selected from a standard deck and replaced. This experiment is repeated a total of five times. Find the probability of selecting exactly three clubs.

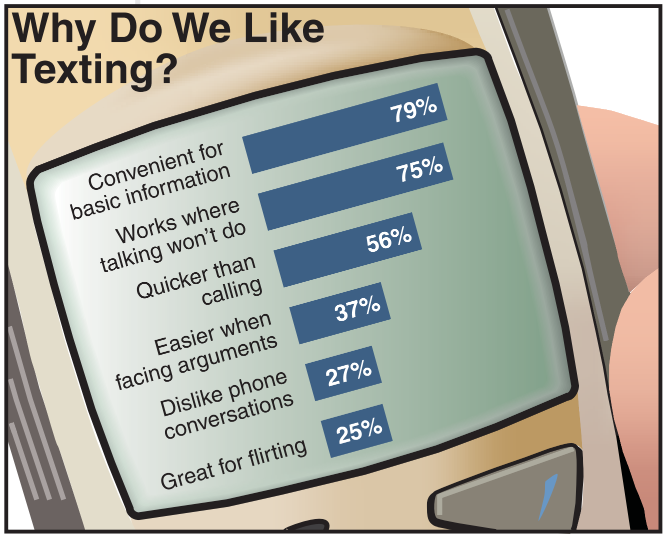
a. *Identify* a trial, a success, and a failure.
b. *Identify* n, p, q, and x.
c. Use the *binomial probability formula*.

Answer: Page A36

By listing the possible values of x with the corresponding probabilities, you can construct a **binomial probability distribution.**

EXAMPLE 3

Constructing a Binomial Distribution



(Source: GfK Roper for Best Buy Mobile)

In a survey, U.S. adults were asked to give reasons why they liked texting on their cellular phones. The results are shown in the graph. Seven adults who participated in the survey are randomly selected and asked whether they like texting because it is quicker than calling. Create a binomial probability distribution for the number of adults who respond yes.

Solution

From the graph, you can see that 56% of adults like texting because it is quicker than calling. So, $p = 0.56$ and $q = 0.44$. Because $n = 7$, the possible values of x are 0, 1, 2, 3, 4, 5, 6, and 7.

$$P(0) = {}_7C_0(0.56)^0(0.44)^7 = 1(0.56)^0(0.44)^7 \approx 0.0032$$
$$P(1) = {}_7C_1(0.56)^1(0.44)^6 = 7(0.56)^1(0.44)^6 \approx 0.0284$$
$$P(2) = {}_7C_2(0.56)^2(0.44)^5 = 21(0.56)^2(0.44)^5 \approx 0.1086$$
$$P(3) = {}_7C_3(0.56)^3(0.44)^4 = 35(0.56)^3(0.44)^4 \approx 0.2304$$
$$P(4) = {}_7C_4(0.56)^4(0.44)^3 = 35(0.56)^4(0.44)^3 \approx 0.2932$$
$$P(5) = {}_7C_5(0.56)^5(0.44)^2 = 21(0.56)^5(0.44)^2 \approx 0.2239$$
$$P(6) = {}_7C_6(0.56)^6(0.44)^1 = 7(0.56)^6(0.44)^1 \approx 0.0950$$
$$P(7) = {}_7C_7(0.56)^7(0.44)^0 = 1(0.56)^7(0.44)^0 \approx 0.0173$$

x	$P(x)$
0	0.0032
1	0.0284
2	0.1086
3	0.2304
4	0.2932
5	0.2239
6	0.0950
7	0.0173
	$\Sigma P(x) = 1$

Notice in the table at the left that all the probabilities are between 0 and 1 and that the sum of the probabilities is 1.

STUDY TIP

When probabilities are rounded to a fixed number of decimal places, the sum of the probabilities may differ slightly from 1.

Try It Yourself 3

Seven adults who participated in the survey are randomly selected and asked whether they like texting because it works where talking won't do. Create a binomial distribution for the number of adults who respond yes.

a. *Identify* a trial, a success, and a failure.
b. *Identify* n, p, q, and possible values for x.
c. Use the *binomial probability formula* for each value of x.
d. Use a *table* to show that the properties of a probability distribution are satisfied.

Answer: Page A37

FINDING BINOMIAL PROBABILITIES

In Examples 2 and 3, you used the binomial probability formula to find the probabilities. A more efficient way to find binomial probabilities is to use a calculator or a computer. For instance, you can find binomial probabilities using MINITAB, Excel, and the TI-83/84 Plus.

EXAMPLE 4

Finding a Binomial Probability Using Technology

The results of a recent survey indicate that 67% of U.S. adults consider air conditioning a necessity. If you randomly select 100 adults, what is the probability that exactly 75 adults consider air conditioning a necessity? Use a technology tool to find the probability. *(Source: Opinion Research Corporation)*

Solution

MINITAB, Excel, and the TI-83/84 Plus each have features that allow you to find binomial probabilities automatically. Try using these technologies. You should obtain results similar to the following.

MINITAB

Probability Distribution Function

Binomial with n = 100 and p = 0.67

x	P(X=x)
75	0.0201004

TI-83/84 PLUS

binompdf(100,.67,75)
.0201004116

EXCEL

	A	B	C	D
1	BINOMDIST(75,100,0.67,FALSE)			
2				0.020100412

Interpretation From these displays, you can see that the probability that exactly 75 adults consider air conditioning a necessity is about 0.02. Because 0.02 is less than 0.05, this can be considered an unusual event.

Try It Yourself 4

The results of a recent survey indicate that 71% of people in the United States use more than one topping on their hot dogs. If you randomly select 250 people, what is the probability that exactly 178 of them will use more than one topping? Use a technology tool to find the probability. *(Source: ICR Survey Research Group for Hebrew International)*

a. *Identify* n, p, and x.
b. Calculate the *binomial probability*.
c. *Interpret* the results.
d. Determine if the event is *unusual*. Explain.

Answer: Page A37

STUDY TIP

Here are instructions for finding a binomial probability on a TI-83/84 Plus.

2nd DISTR

0: binompdf(

Enter the values of n, p, and x separated by commas.

ENTER

STUDY TIP

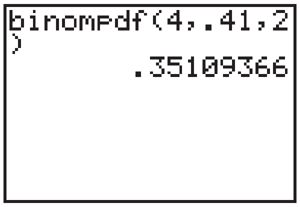
Recall that if a probability is 0.05 or less, it is typically considered unusual.

EXAMPLE 5

SC Report 18

Finding Binomial Probabilities Using Formulas

A survey indicates that 41% of women in the United States consider reading their favorite leisure-time activity. You randomly select four U.S. women and ask them if reading is their favorite leisure-time activity. Find the probability that (1) exactly two of them respond yes, (2) at least two of them respond yes, and (3) fewer than two of them respond yes. *(Source: Louis Harris & Associates)*



Using a TI-83/84 Plus, you can find the probability in part (1) automatically.

Solution

1. Using $n = 4$, $p = 0.41$, $q = 0.59$, and $x = 2$, the probability that exactly two women will respond yes is

$$P(2) = {}_4C_2(0.41)^2(0.59)^2 = 6(0.41)^2(0.59)^2 \approx 0.351.$$

2. To find the probability that at least two women will respond yes, find the sum of $P(2)$, $P(3)$, and $P(4)$.

$$P(2) = {}_4C_2(0.41)^2(0.59)^2 = 6(0.41)^2(0.59)^2 \approx 0.351094$$
$$P(3) = {}_4C_3(0.41)^3(0.59)^1 = 4(0.41)^3(0.59)^1 \approx 0.162654$$
$$P(4) = {}_4C_4(0.41)^4(0.59)^0 = 1(0.41)^4(0.59)^0 \approx 0.028258$$

So, the probability that at least two will respond yes is

$$\begin{aligned} P(x \geq 2) &= P(2) + P(3) + P(4) \\ &\approx 0.351094 + 0.162654 + 0.028258 \\ &\approx 0.542. \end{aligned}$$

STUDY TIP

The complement of "x is at least 2" is "x is less than 2." So, another way to find the probability in part (3) is

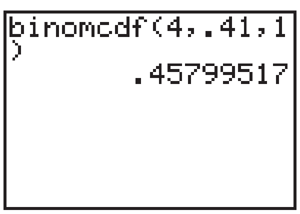
$$\begin{aligned} P(x < 2) &= 1 - P(x \geq 2) \\ &\approx 1 - 0.542 \\ &= 0.458. \end{aligned}$$

3. To find the probability that fewer than two women will respond yes, find the sum of $P(0)$ and $P(1)$.

$$P(0) = {}_4C_0(0.41)^0(0.59)^4 = 1(0.41)^0(0.59)^4 \approx 0.121174$$
$$P(1) = {}_4C_1(0.41)^1(0.59)^3 = 4(0.41)^1(0.59)^3 \approx 0.336822$$

So, the probability that fewer than two will respond yes is

$$\begin{aligned} P(x < 2) &= P(0) + P(1) \\ &\approx 0.121174 + 0.336822 \\ &\approx 0.458. \end{aligned}$$



The cumulative distribution function (CDF) computes the probability of "x or fewer" successes. The CDF adds the areas for the given x-value and all those to its left.

Try It Yourself 5

A survey indicates that 21% of men in the United States consider fishing their favorite leisure-time activity. You randomly select five U.S. men and ask them if fishing is their favorite leisure-time activity. Find the probability that (1) exactly two of them respond yes, (2) at least two of them respond yes, and (3) fewer than two of them respond yes. *(Source: Louis Harris & Associates)*

a. Determine the appropriate *value of x* for each situation.
b. Find the *binomial probability* for each value of x. Then find the *sum*, if necessary.
c. *Write* the result as a sentence.

Answer: Page A37

Finding binomial probabilities with the binomial probability formula can be a tedious process. To make this process easier, you can use a binomial probability table. Table 2 in Appendix B lists the binomial probabilities for selected values of n and p.

EXAMPLE 6

Finding a Binomial Probability Using a Table

About ten percent of workers (16 years and over) in the United States commute to their jobs by carpooling. You randomly select eight workers. What is the probability that exactly four of them carpool to work? Use a table to find the probability. *(Source: American Community Survey)*

Solution

A portion of Table 2 in Appendix B is shown here. Using the distribution for $n = 8$ and $p = 0.1$, you can find the probability that $x = 4$, as shown by the highlighted areas in the table.

		p												
n	***x***	**.01**	**.05**	**.10**	**.15**	**.20**	**.25**	**.30**	**.35**	**.40**	**.45**	**.50**	**.55**	**.60**
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216
8	0	.923	.663	.430	.272	.168	.100	.058	.032	.017	.008	.004	.002	.001
	1	.075	.279	.383	.385	.336	.267	.198	.137	.090	.055	.031	.016	.008
	2	.003	.051	.149	.238	.294	.311	.296	.259	.209	.157	.109	.070	.041
	3	.000	.005	.033	.084	.147	.208	.254	.279	.279	.257	.219	.172	.124
	4	.000	.000	.005	.018	.046	.087	.136	.188	.232	.263	.273	.263	.232
	5	.000	.000	.000	.003	.009	.023	.047	.081	.124	.172	.219	.257	.279
	6	.000	.000	.000	.000	.001	.004	.010	.022	.041	.070	.109	.157	.209
	7	.000	.000	.000	.000	.000	.000	.001	.003	.008	.016	.031	.055	.090
	8	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.017

To explore this topic further, see Activity 4.2 on page 216.

Interpretation So, the probability that exactly four of the eight workers carpool to work is 0.005. Because 0.005 is less than 0.05, this can be considered an unusual event.

Try It Yourself 6

About fifty-five percent of all small businesses in the United States have a website. If you randomly select 10 small businesses, what is the probability that exactly four of them have a website? Use a table to find the probability. *(Adapted from Webvisible/Nielsen Online)*

a. *Identify* a trial, a success, and a failure.
b. Identify n, p, and x.
c. Use Table 2 in Appendix B to find the *binomial probability*.
d. *Interpret* the results.
e. Determine if the event is *unusual*. Explain.

Answer: Page A37

GRAPHING BINOMIAL DISTRIBUTIONS

In Section 4.1, you learned how to graph discrete probability distributions. Because a binomial distribution is a discrete probability distribution, you can use the same process.

EXAMPLE 7

Graphing a Binomial Distribution

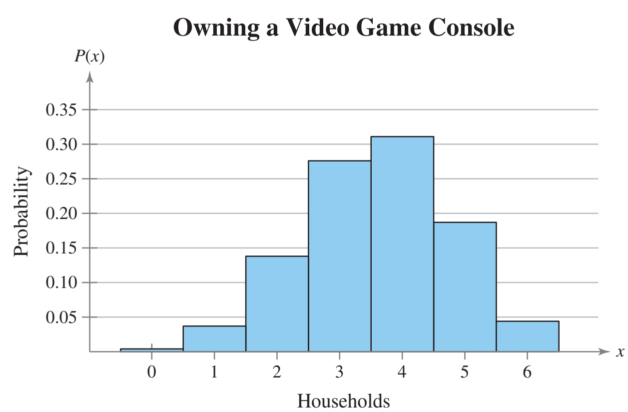
Sixty percent of households in the United States own a video game console. You randomly select six households and ask them if they own a video game console. Construct a probability distribution for the random variable x. Then graph the distribution. *(Source: Deloitte LLP)*

Solution

To construct the binomial distribution, find the probability for each value of x. Using $n = 6$, $p = 0.6$, and $q = 0.4$, you can obtain the following.

x	0	1	2	3	4	5	6
$P(x)$	0.004	0.037	0.138	0.276	0.311	0.187	0.047

You can graph the probability distribution using a histogram as shown below.



Interpretation From the histogram, you can see that it would be unusual if none, only one, or all six of the households owned a video game console because of the low probabilities.

Try It Yourself 7

Eighty-one percent of households in the United States own a computer. You randomly select four households and ask if they own a computer. Construct a probability distribution for the random variable x. Then graph the distribution. *(Source: Nielsen)*

a. Find the *binomial probability* for each value of the random variable x.
b. *Organize* the values of x and their corresponding probabilities in a table.
c. Use a *histogram* to graph the binomial distribution. Then describe its shape.
d. Are any of the events *unusual*? Explain. *Answer: Page A37*

Notice in Example 7 that the histogram is skewed left. The graph of a binomial distribution with $p > 0.5$ is skewed left, whereas the graph of a binomial distribution with $p < 0.5$ is skewed right. The graph of a binomial distribution with $p = 0.5$ is symmetric.

MEAN, VARIANCE, AND STANDARD DEVIATION

Although you can use the formulas you learned in Section 4.1 for mean, variance, and standard deviation of a discrete probability distribution, the properties of a binomial distribution enable you to use much simpler formulas.

POPULATION PARAMETERS OF A BINOMIAL DISTRIBUTION

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard deviation: $\sigma = \sqrt{npq}$

EXAMPLE 8

Finding and Interpreting Mean, Variance, and Standard Deviation

In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days during the month of June. Interpret the results and determine any unusual values. *(Source: National Climatic Data Center)*

Solution

There are 30 days in June. Using $n = 30$, $p = 0.56$, and $q = 0.44$, you can find the mean, variance, and standard deviation as shown below.

$$\mu = np = 30 \cdot 0.56 = 16.8$$

$$\sigma^2 = npq = 30 \cdot 0.56 \cdot 0.44 \approx 7.4$$

$$\sigma = \sqrt{npq} = \sqrt{30 \cdot 0.56 \cdot 0.44} \approx 2.7$$

Interpretation On average, there are 16.8 cloudy days during the month of June. The standard deviation is about 2.7 days. Values that are more than two standard deviations from the mean are considered unusual. Because $16.8 - 2(2.7) = 11.4$, a June with 11 cloudy days or less would be unusual. Similarly, because $16.8 + 2(2.7) = 22.2$, a June with 23 cloudy days or more would also be unusual.

Try It Yourself 8

In San Francisco, California, 44% of the days in a year are clear. Find the mean, variance, and standard deviation for the number of clear days during the month of May. Interpret the results and determine any unusual events. *(Source: National Climatic Data Center)*

a. *Identify* a success and the values of n, p, and q.
b. Find the *product* of n and p to calculate the mean.
c. Find the *product* of n, p, and q for the variance.
d. Find the *square root* of the variance to find the standard deviation.
e. *Interpret* the results.
f. Determine any *unusual* events.

Answer: Page A37

4.2 EXERCISES

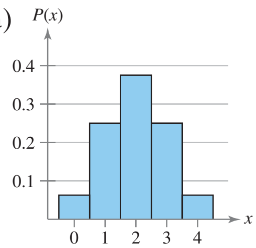
BUILDING BASIC SKILLS AND VOCABULARY

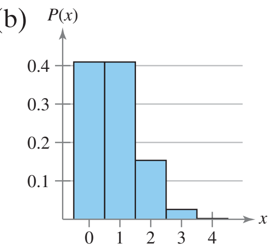
1. In a binomial experiment, what does it mean to say that each trial is independent of the other trials?

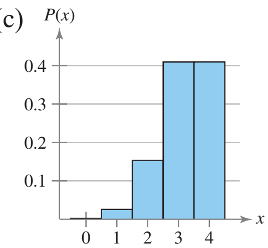
2. In a binomial experiment with n trials, what does the random variable measure?

Graphical Analysis *In Exercises 3 and 4, match each given probability with the correct graph. The histograms represent binomial distributions. Each distribution has the same number of trials n but different probabilities of success p.*

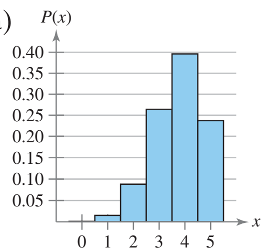
3. $p = 0.20,\ p = 0.50,\ p = 0.80$

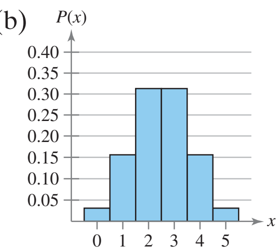
(a)


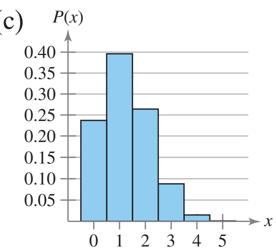
(b)


(c)


4. $p = 0.25,\ p = 0.50,\ p = 0.75$

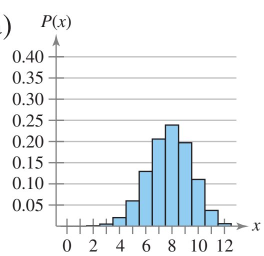
(a)


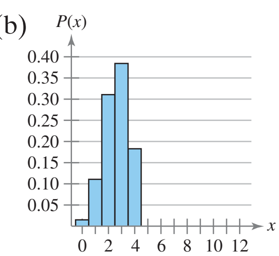
(b)


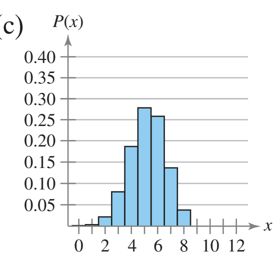
(c)


Graphical Analysis *In Exercises 5 and 6, match each given value of n with the correct graph. Each histogram shown represents part of a binomial distribution. Each distribution has the same probability of success p but different numbers of trials n. What happens as the value of n increases and p remains the same?*

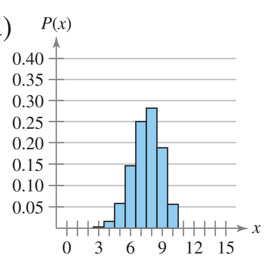
5. $n = 4,\ n = 8,\ n = 12$

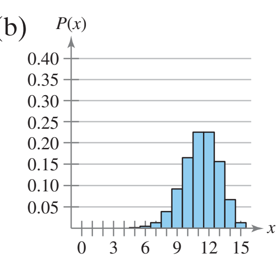
(a)


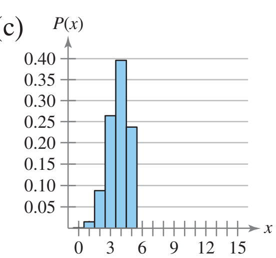
(b)


(c)


6. $n = 5,\ n = 10,\ n = 15$

(a)


(b)


(c)


7. Identify the unusual values of x in each histogram in Exercise 5.

8. Identify the unusual values of x in each histogram in Exercise 6.

Identifying and Understanding Binomial Experiments *In Exercises 9–12, decide whether the experiment is a binomial experiment. If it is, identify a success, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not a binomial experiment, explain why.*

9. Cyanosis Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies born with cyanosis recover fully. A hospital is caring for five babies born with cyanosis. The random variable represents the number of babies that recover fully. *(Source: The World Book Encyclopedia)*

10. Clothing Store Purchases From past records, a clothing store finds that 26% of the people who enter the store will make a purchase. During a one-hour period, 18 people enter the store. The random variable represents the number of people who do not make a purchase.

11. Survey A survey asks 1400 chief financial officers, "Has the economy forced you to postpone or reduce the amount of vacation you plan to take this year?" Thirty-one percent of those surveyed say they are postponing or reducing the amount of vacation. Twenty officers participating in the survey are randomly selected. The random variable represents the number of officers who are postponing or reducing the amount of vacation. *(Source: Robert Half Management Resources)*

12. Lottery A state lottery randomly chooses 6 balls numbered from 1 through 40. You choose six numbers and purchase a lottery ticket. The random variable represents the number of matches on your ticket to the numbers drawn in the lottery.

Mean, Variance, and Standard Deviation *In Exercises 13–16, find the mean, variance, and standard deviation of the binomial distribution with the given values of n and p.*

13. $n = 50, p = 0.4$

14. $n = 84, p = 0.65$

15. $n = 124, p = 0.26$

16. $n = 316, p = 0.82$

USING AND INTERPRETING CONCEPTS

Finding Binomial Probabilities *In Exercises 17–26, find the indicated probabilities. If convenient, use technology to find the probabilities.*

17. Answer Guessing You are taking a multiple-choice quiz that consists of five questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. Find the probability of guessing (a) exactly three answers correctly, (b) at least three answers correctly, and (c) less than three answers correctly.

18. Surgery Success A surgical technique is performed on seven patients. You are told there is a 70% chance of success. Find the probability that the surgery is successful for (a) exactly five patients, (b) at least five patients, and (c) less than five patients.

19. Baseball Fans Fifty-nine percent of men consider themselves fans of professional baseball. You randomly select 10 men and ask each if he considers himself a fan of professional baseball. Find the probability that the number who consider themselves baseball fans is (a) exactly eight, (b) at least eight, and (c) less than eight. *(Source: Gallup Poll)*

20. Favorite Cookie Ten percent of adults say oatmeal raisin is their favorite cookie. You randomly select 12 adults and ask them to name their favorite cookie. Find the probability that the number who say oatmeal raisin is their favorite cookie is (a) exactly four, (b) at least four, and (c) less than four. *(Source: WEAREVER)*

21. Savings Fifty-five percent of U.S. households say they would feel secure if they had $50,000 in savings. You randomly select 8 households and ask them if they would feel secure if they had $50,000 in savings. Find the probability that the number that say they would feel secure is (a) exactly five, (b) more than five, and (c) at most five. *(Source: HSBC Consumer Survey)*

22. Honeymoon Financing Seventy percent of married couples paid for their honeymoon themselves. You randomly select 20 married couples and ask them if they paid for their honeymoon themselves. Find the probability that the number of couples who say they paid for their honeymoon themselves is (a) exactly one, (b) more than one, and (c) at most one. *(Source: Bride's Magazine)*

23. Financial Advice Forty-three percent of adults say they get their financial advice from family members. You randomly select 14 adults and ask them if they get their financial advice from family members. Find the probability that the number who say they get their financial advice from family members is (a) exactly five, (b) at least six, and (c) at most three. *(Source: Sun Life Unretirement Index)*

24. Retirement Fourteen percent of workers believe they will need less than $250,000 when they retire. You randomly select 10 workers and ask them how much money they think they will need for retirement. Find the probability that the number of workers who say they will need less than $250,000 when they retire is (a) exactly two, (b) more than six, and (c) at most five. *(Source: Retirement Corporation of America)*

25. Credit Cards Twenty-eight percent of college students say they use credit cards because of the rewards program. You randomly select 10 college students and ask them to name the reason they use credit cards. Find the probability that the number of college students who say they use credit cards because of the rewards program is (a) exactly two, (b) more than two, and (c) between two and five, inclusive. *(Source: Experience.com)*

26. Movies on Phone Twenty-five percent of adults say they would watch streaming movies on their phone at work. You randomly select 12 adults and ask them if they would watch streaming movies on their phone at work. Find the probability that the number who say they would watch streaming movies on their phone at work is (a) exactly four, (b) more than four, and (c) between four and eight, inclusive. *(Source: mSpot)*

Constructing Binomial Distributions *In Exercises 27–30, (a) construct a binomial distribution, (b) graph the binomial distribution using a histogram and describe its shape, (c) find the mean, variance, and standard deviation of the binomial distribution, and (d) interpret the results in the context of the real-life situation. What values of the random variable x would you consider unusual? Explain your reasoning.*

27. Visiting the Dentist Sixty-three percent of adults say they are visiting the dentist less because of the economy. You randomly select six adults and ask them if they are visiting the dentist less because of the economy. *(Source: American Optometric Association)*

28. No Trouble Sleeping One in four adults claims to have no trouble sleeping at night. You randomly select five adults and ask them if they have any trouble sleeping at night. *(Source: Marist Institute for Public Opinion)*

29. Blood Donors Five percent of people in the United States eligible to donate blood actually do. You randomly select four eligible blood donors and ask them if they donate blood. *(Source: MetLife Consumer Education Center)*

30. Blood Types Thirty-nine percent of people in the United States have type O^+ blood. You randomly select five Americans and ask them if their blood type is O^+. *(Source: American Association of Blood Banks)*

31. Annoying Flights The graph shows the results of a survey of travelers who were asked to name what they found most annoying on a flight. You randomly select six people who participated in the survey and ask them to name what they find most annoying on a flight. Let x represent the number who name crying kids as the most annoying. *(Source: USA Today)*

(a) Construct a binomial distribution.

(b) Find the probability that exactly two people name "crying kids."

(c) Find the probability that at least five people name "crying kids."

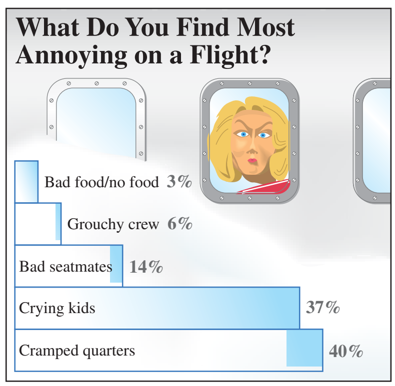


FIGURE FOR EXERCISE 31

FIGURE FOR EXERCISE 32

32. Small-Business Owners The graph shows the results of a survey of small-business owners who were asked which business skills they would like to develop further. You randomly select five owners who participated in the survey and ask them which business skills they want to develop further. Let x represent the number who said financial management was the skill they wanted to develop further. *(Source: American Express)*

(a) Construct a binomial distribution.

(b) Find the probability that exactly two owners say "financial management."

(c) Find the probability that fewer than four owners say "financial management."

33. Find the mean and standard deviation of the binomial distribution in Exercise 31 and interpret the results in the context of the real-life situation. What values of x would you consider unusual? Explain your reasoning.

34. Find the mean and standard deviation of the binomial distribution in Exercise 32 and interpret the results in the context of the real-life situation. What values of x would you consider unusual? Explain your reasoning.

SC *In Exercises 35 and 36, use the StatCrunch binomial calculator to find the indicated probabilities. Then determine if the event is unusual. Explain your reasoning.*

35. Pet Owners Sixty-six percent of pet owners say they consider their pet to be their best friend. You randomly select 10 pet owners and ask them if they consider their pet to be their best friend. Find the probability that the number who say their pet is their best friend is (a) exactly nine, (b) at least seven, and (c) at most three. *(Adapted from Kelton Research)*

36. Eco-Friendly Vehicles Fifty-three percent of 18- to 30-year-olds say they would pay more for an eco-friendly vehicle. You randomly select eight 18- to 30-year-olds and ask each if they would pay more for an eco-friendly vehicle. Find the probability that the number who say they would pay more for an eco-friendly vehicle is (a) exactly four, (b) at least five, and (c) less than two. *(Source: Deloitte LLP and Michigan State University)*

EXTENDING CONCEPTS

Multinomial Experiments *In Exercises 37 and 38, use the following information.*

A **multinomial experiment** is a probability experiment that satisfies the following conditions.

1. The experiment is repeated a fixed number of times n where each trial is independent of the other trials.
2. Each trial has k possible mutually exclusive outcomes: $E_1, E_2, E_3, \ldots, E_k$.
3. Each outcome has a fixed probability. So, $P(E_1) = p_1$, $P(E_2) = p_2$, $P(E_3) = p_3, \ldots, P(E_k) = p_k$. The sum of the probabilities for all outcomes is
$$p_1 + p_2 + p_3 + \cdots + p_k = 1.$$
4. x_1 is the number of times E_1 will occur, x_2 is the number of times E_2 will occur, x_3 is the number of times E_3 will occur, and so on.
5. The discrete random variable x counts the number of times $x_1, x_2, x_3, \ldots, x_k$ occurs in n independent trials where
$$x_1 + x_2 + x_3 + \cdots + x_k = n.$$
The probability that x will occur is
$$P(x) = \frac{n!}{x_1!x_2!x_3! \cdots x_k!} p_1^{x_1} p_2^{x_2} p_3^{x_3} \ldots p_k^{x_k}.$$

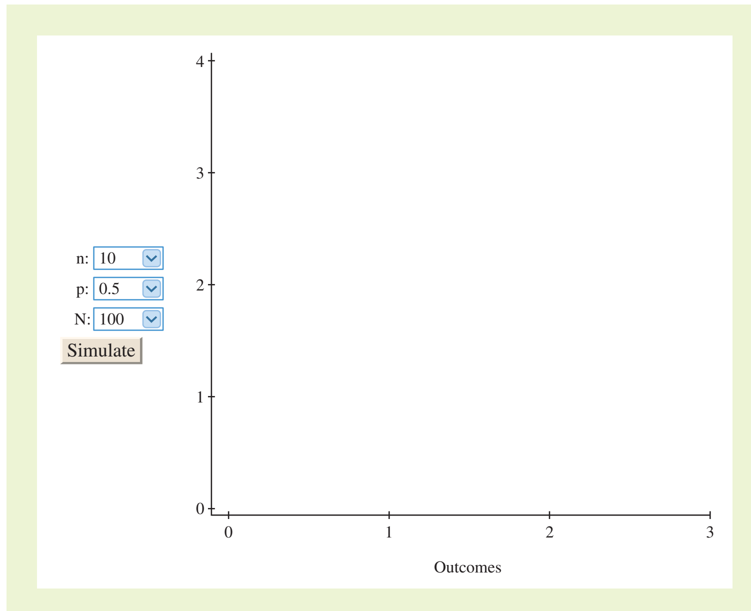
37. Genetics According to a theory in genetics, if tall and colorful plants are crossed with short and colorless plants, four types of plants will result: tall and colorful, tall and colorless, short and colorful, and short and colorless, with corresponding probabilities of $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$. If 10 plants are selected, find the probability that 5 will be tall and colorful, 2 will be tall and colorless, 2 will be short and colorful, and 1 will be short and colorless.

38. Genetics Another proposed theory in genetics gives the corresponding probabilities for the four types of plants described in Exercise 37 as $\frac{5}{16}$, $\frac{4}{16}$, $\frac{1}{16}$, and $\frac{6}{16}$. If 10 plants are selected, find the probability that 5 will be tall and colorful, 2 will be tall and colorless, 2 will be short and colorful, and 1 will be short and colorless.

ACTIVITY 4.2 Binomial Distribution

APPLET

The *binomial distribution* applet allows you to simulate values from a binomial distribution. You can specify the parameters for the binomial distribution (n and p) and the number of values to be simulated (N). When you click SIMULATE, N values from the specified binomial distribution will be plotted at the right. The frequency of each outcome is shown in the plot.



Explore

Step 1 Specify a value of n.
Step 2 Specify a value of p.
Step 3 Specify a value of N.
Step 4 Click SIMULATE.

Draw Conclusions

APPLET

1. During a presidential election year, 70% of a county's eligible voters actually vote. Simulate selecting $n = 10$ eligible voters $N = 10$ times (for 10 communities in the county). Use the results to estimate the probability that the number who voted in this election is (a) exactly 5, (b) at least 8, and (c) at most 7.

2. During a non-presidential election year, 20% of the eligible voters in the same county as in Exercise 1 actually vote. Simulate selecting $n = 10$ eligible voters $N = 10$ times (for 10 communities in the county). Use the results to estimate the probability that the number who voted in this election is (a) exactly 4, (b) at least 5, and (c) less than 4.

3. Suppose in Exercise 1 you select $n = 10$ eligible voters $N = 100$ times. Estimate the probability that the number who voted in this election is exactly 5. Compare this result with the result in Exercise 1 part (a). Which of these is closer to the probability found using the binomial probability formula?

Binomial Distribution of Airplane Accidents

The Air Transport Association of America (ATA) is a support organization for the principal U.S. airlines. Some of the ATA's activities include promoting the air transport industry and conducting industry-wide studies.

The ATA also keeps statistics about commercial airline flights, including those that involve accidents. From 1979 through 2008 for aircraft with 10 or more seats, there were 76 fatal commercial airplane accidents involving U.S. airlines. The distribution of these accidents is shown in the histogram at the right.

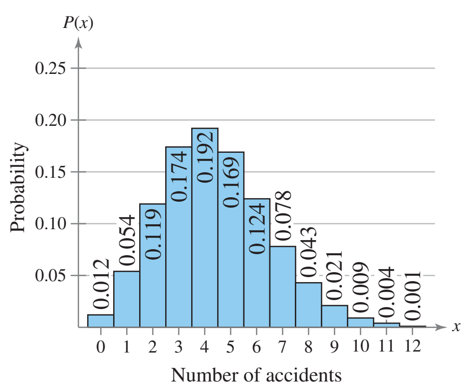
Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Accidents	4	0	4	4	4	1	4	2	4	3	5	4	3	3	1

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Accidents	4	1	3	3	1	2	2	6	0	2	1	3	2	0	0

EXERCISES

1. In 2006, there were about 11 million commercial flights in the United States. If one is selected at random, what is the probability that it involved a fatal accident?

2. Suppose that the probability of a fatal accident in a given year is 0.0000004. A binomial probability distribution for $n = 11{,}000{,}000$ and $p = 0.0000004$ with $x = 0$ to 12 is shown.



What is the probability that there will be (a) 4 fatal accidents in a year? (b) 10 fatal accidents? (c) between 1 and 5, inclusive?

3. Construct a binomial distribution for $n = 11{,}000{,}000$ and $p = 0.0000008$ with $x = 0$ to 12. Compare your results with the distribution in Exercise 2.

4. Is a binomial distribution a good model for determining the probabilities of various numbers of fatal accidents during a year? Explain your reasoning and include a discussion of the four criteria for a binomial experiment.

5. According to analysis by *USA TODAY*, air flight is so safe that a person "would have to fly every day for more than 64,000 years before dying in an accident." How can such a statement be justified?

4.3 More Discrete Probability Distributions

WHAT YOU SHOULD LEARN

- How to find probabilities using the geometric distribution
- How to find probabilities using the Poisson distribution

The Geometric Distribution ▸ The Poisson Distribution ▸ Summary of Discrete Probability Distributions

▸ THE GEOMETRIC DISTRIBUTION

In this section, you will study two more discrete probability distributions—the geometric distribution and the Poisson distribution.

Many actions in life are repeated until a success occurs. For instance, a CPA candidate might take the CPA exam several times before receiving a passing score, or you might have to send an e-mail several times before it is successfully sent. Situations such as these can be represented by a *geometric distribution*.

DEFINITION

A **geometric distribution** is a discrete probability distribution of a random variable x that satisfies the following conditions.

1. A trial is repeated until a success occurs.
2. The repeated trials are independent of each other.
3. The probability of success p is constant for each trial.
4. The random variable x represents the number of the trial in which the first success occurs.

The probability that the first success will occur on trial number x is

$$P(x) = pq^{x-1}, \text{ where } q = 1 - p.$$

In other words, when the first success occurs on the third trial, the outcome is *FFS*, and the probability is $P(3) = q \cdot q \cdot p$, or $P(3) = p \cdot q^2$.

STUDY TIP

Here are instructions for finding a geometric probability on a TI-83/84 Plus.

2nd DISTR

D: geometpdf(

Enter the values of p and x separated by commas.

ENTER

EXAMPLE 1

▸ Finding Probabilities Using the Geometric Distribution

Basketball player LeBron James makes a free throw shot about 74% of the time. Find the probability that the first free throw shot LeBron makes occurs on the third or fourth attempt. *(Source: ESPN)*

▸ Solution To find the probability that LeBron makes his first free throw shot on the third or fourth attempt, first find the probability that the first shot he makes will occur on the third attempt and the probability that the first shot he makes will occur on the fourth attempt. Then, find the sum of the resulting probabilities. Using $p = 0.74$, $q = 0.26$, and $x = 3$, you have

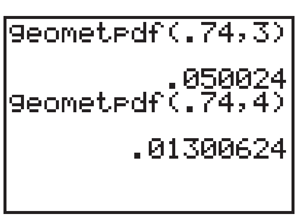
$$P(3) = 0.74 \cdot (0.26)^2 = 0.050024.$$

Using $p = 0.74$, $q = 0.26$, and $x = 4$, you have

$$P(4) = 0.74 \cdot (0.26)^3 \approx 0.013006.$$

So, the probability that LeBron makes his first free throw shot on the third or fourth attempt is

$$P(\text{shot made on third or fourth attempt}) = P(3) + P(4) \approx 0.050024 + 0.013006 \approx 0.063.$$



Using a TI-83/84 Plus, you can find the probabilities used in Example 1 automatically.

▸ Try It Yourself 1

Find the probability that LeBron makes his first free throw shot before his third attempt.

a. Use the *geometric distribution* to find $P(1)$ and $P(2)$.
b. Find the *sum* of $P(1)$ and $P(2)$.
c. *Write* the result as a sentence.

Answer: Page A37

Even though theoretically a success may never occur, the geometric distribution is a discrete probability distribution because the values of x can be listed—1, 2, 3, ... Notice that as x becomes larger, $P(x)$ gets closer to zero. For instance,

$$P(15) = 0.74(0.26)^{14} \approx 0.0000000048.$$

▸ THE POISSON DISTRIBUTION

In a binomial experiment, you are interested in finding the probability of a specific number of successes in a given number of trials. Suppose instead that you want to know the probability that a specific number of occurrences takes place within a given unit of time or space. For instance, to determine the probability that an employee will take 15 sick days within a year, you can use the *Poisson distribution*.

STUDY TIP

Here are instructions for finding a Poisson probability on a TI-83/84 Plus.

2nd DISTR
B: poissonpdf(

Enter the values of μ and x separated by commas.

ENTER

DEFINITION

The **Poisson distribution** is a discrete probability distribution of a random variable x that satisfies the following conditions.

1. The experiment consists of counting the number of times x an event occurs in a given interval. The interval can be an interval of time, area, or volume.
2. The probability of the event occurring is the same for each interval.
3. The number of occurrences in one interval is independent of the number of occurrences in other intervals.

The probability of exactly x occurrences in an interval is

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where e is an irrational number approximately equal to 2.71828 and μ is the mean number of occurrences per interval unit.

Using a TI-83/84 Plus, you can find the probability in Example 2 automatically.

EXAMPLE 2

SC Report 19

▸ Using the Poisson Distribution

The mean number of accidents per month at a certain intersection is three. What is the probability that in any given month four accidents will occur at this intersection?

▸ Solution

Using $x = 4$ and $\mu = 3$, the probability that 4 accidents will occur in any given month at the intersection is

$$P(4) \approx \frac{3^4(2.71828)^{-3}}{4!} \approx 0.168.$$

▸ Try It Yourself 2

What is the probability that more than four accidents will occur in any given month at the intersection?

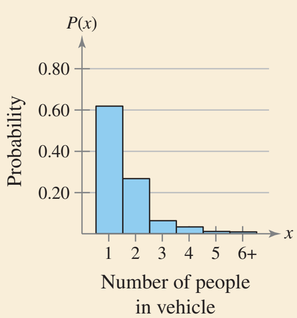
a. Use the *Poisson distribution* to find $P(0)$, $P(1)$, $P(2)$, $P(3)$, and $P(4)$.
b. Find the *sum* of $P(0)$, $P(1)$, $P(2)$, $P(3)$, and $P(4)$.
c. *Subtract* the sum from 1.
d. *Write* the result as a sentence.

Answer: Page A37

In Example 2, you used a formula to determine a Poisson probability. You can also use a table to find Poisson probabilities. Table 3 in Appendix B lists the Poisson probabilities for selected values of x and μ. You can use technology tools, such as MINITAB, Excel, and the TI-83/84 Plus, to find Poisson probabilities as well.

PICTURING THE WORLD

The first successful suspension bridge built in the United States, the Tacoma Narrows Bridge, spans the Tacoma Narrows in Washington State. The average occupancy of vehicles that travel across the bridge is 1.6. The following probability distribution represents the vehicle occupancy on the bridge during a five-day period. (Adapted from Washington State Department of Transportation)



What is the probability that a randomly selected vehicle has two occupants or fewer?

EXAMPLE 3

▸ Finding Poisson Probabilities Using a Table

A population count shows that the average number of rabbits per acre living in a field is 3.6. Use a table to find the probability that seven rabbits are found on any given acre of the field.

▸ Solution

A portion of Table 3 in Appendix B is shown here. Using the distribution for $\mu = 3.6$ and $x = 7$, you can find the Poisson probability as shown by the highlighted areas in the table.

	μ						
x	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**	**3.6**	**3.7**
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087
4	.1734	.1781	.1823	.1858	.1888	.1912	.1931
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033

Interpretation So, the probability that seven rabbits are found on any given acre is 0.0425. Because 0.0425 is less than 0.05, this can be considered an unusual event.

▸ Try It Yourself 3

Two thousand brown trout are introduced into a small lake. The lake has a volume of 20,000 cubic meters. Use a table to find the probability that three brown trout are found in any given cubic meter of the lake.

a. Find the *average* number of brown trout per cubic meter.
b. *Identify* μ and x.
c. *Use* Table 3 in Appendix B to find the Poisson probability.
d. *Interpret* the results.
e. Determine if the event is *unusual*. Explain.

Answer: Page A37

SUMMARY OF DISCRETE PROBABILITY DISTRIBUTIONS

The following table summarizes the discrete probability distributions discussed in this chapter.

Distribution	Summary	Formulas
Binomial Distribution	A binomial experiment satisfies the following conditions. **1.** The experiment is repeated for a fixed number n of independent trials. **2.** There are only two possible outcomes for each trial. Each outcome can be classified as a success or as a failure. **3.** The probability of a success must remain constant for each trial. **4.** The random variable x counts the number of successful trials. The parameters of a binomial distribution are n and p.	n = the number of times a trial repeats x = the number of successes in n trials p = probability of success in a single trial q = probability of failure in a single trial $q = 1 - p$ The probability of exactly x successes in n trials is $P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}.$
Geometric Distribution	A geometric distribution is a discrete probability distribution of a random variable x that satisfies the following conditions. **1.** A trial is repeated until a success occurs. **2.** The repeated trials are independent of each other. **3.** The probability of success p is constant for each trial. **4.** The random variable x represents the number of the trial in which the first success occurs. The parameter of a geometric distribution is p.	x = the number of the trial in which the first success occurs p = probability of success in a single trial q = probability of failure in a single trial $q = 1 - p$ The probability that the first success occurs on trial number x is $P(x) = pq^{x-1}.$
Poisson Distribution	The Poisson distribution is a discrete probability distribution of a random variable x that satisfies the following conditions. **1.** The experiment consists of counting the number of times x an event occurs over a specified interval of time, area, or volume. **2.** The probability of the event occurring is the same for each interval. **3.** The number of occurrences in one interval is independent of the number of occurrences in other intervals. The parameter of a Poisson distribution is μ.	x = the number of occurrences in the given interval μ = the mean number of occurrences in a given time or space unit The probability of exactly x occurrences in an interval is $P(x) = \frac{\mu^x e^{-\mu}}{x!}.$

4.3 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

In Exercises 1–4, assume the geometric distribution applies. Use the given probability of success p to find the indicated probability.

1. Find $P(3)$ when $p = 0.65$.

2. Find $P(1)$ when $p = 0.45$.

3. Find $P(5)$ when $p = 0.09$.

4. Find $P(8)$ when $p = 0.28$.

In Exercises 5–8, assume the Poisson distribution applies. Use the given mean μ to find the indicated probability.

5. Find $P(4)$ when $\mu = 5$.

6. Find $P(3)$ when $\mu = 6$.

7. Find $P(2)$ when $\mu = 1.5$.

8. Find $P(5)$ when $\mu = 9.8$.

9. In your own words, describe the difference between the value of x in a binomial distribution and in a geometric distribution.

10. In your own words, describe the difference between the value of x in a binomial distribution and in a Poisson distribution.

Deciding on a Distribution *In Exercises 11–14, decide which probability distribution—binomial, geometric, or Poisson—applies to the question. You do not need to answer the question. Instead, justify your choice.*

11. Pilot's Test *Given:* The probability that a student passes the written test for a private pilot's license is 0.75. *Question:* What is the probability that a student will fail on the first attempt and pass on the second attempt?

12. Precipitation *Given:* In Tampa, Florida, the mean number of days in July with 0.01 inch or more precipitation is 16. *Question:* What is the probability that Tampa has 20 days with 0.01 inch or more precipitation next July? *(Source: National Climatic Data Center)*

13. Carry-On Luggage *Given:* Fifty-four percent of U.S. adults think Congress should place size limits on carry-on bags. In a survey of 110 randomly chosen adults, people are asked, "Do you think Congress should place size limits on carry-on bags?" *Question:* What is the probability that exactly 60 of the people answer yes? *(Source: TripAdvisor)*

14. Breaking Up *Given:* Twenty-nine percent of Americans ages 16 to 21 years old say that they would break up with their boyfriend/girlfriend for $10,000. You select at random twenty 16- to 21-year-olds. *Question:* What is the probability that the first person who says he or she would break up with their boyfriend/girlfriend for $10,000 is the fifth person selected? *(Source: Bank of America Student Banking & Seventeen)*

USING AND INTERPRETING CONCEPTS

Using a Distribution to Find Probabilities *In Exercises 15–22, find the indicated probabilities using the geometric distribution or the Poisson distribution. Then determine if the events are unusual. If convenient, use a Poisson probability table or technology to find the probabilities.*

15. Telephone Sales Assume the probability that you will make a sale on any given telephone call is 0.19. Find the probability that you (a) make your first sale on the fifth call, (b) make your first sale on the first, second, or third call, and (c) do not make a sale on the first three calls.

16. Bankruptcies The mean number of bankruptcies filed per minute in the United States in a recent year was about two. Find the probability that (a) exactly five businesses will file bankruptcy in any given minute, (b) at least five businesses will file bankruptcy in any given minute, and (c) more than five businesses will file bankruptcy in any given minute. *(Source: Administrative Office of the U.S. Courts)*

17. Typographical Errors A newspaper finds that the mean number of typographical errors per page is four. Find the probability that (a) exactly three typographical errors are found on a page, (b) at most three typographical errors are found on a page, and (c) more than three typographical errors are found on a page.

18. Pass Completions Football player Peyton Manning completes a pass 64.8% of the time. Find the probability that (a) the first pass Peyton completes is the second pass, (b) the first pass Peyton completes is the first or second pass, and (c) Peyton does not complete his first two passes. *(Source: National Football League)*

19. Major Hurricanes A major hurricane is a hurricane with wind speeds of 111 miles per hour or greater. During the 20th century, the mean number of major hurricanes to strike the U.S. mainland per year was about 0.6. Find the probability that in a given year (a) exactly one major hurricane strikes the U.S. mainland, (b) at most one major hurricane strikes the U.S. mainland, and (c) more than one major hurricane strikes the U.S. mainland. *(Source: National Hurricane Center)*

20. Glass Manufacturer A glass manufacturer finds that 1 in every 500 glass items produced is warped. Find the probability that (a) the first warped glass item is the tenth item produced, (b) the first warped glass item is the first, second, or third item produced, and (c) none of the first 10 glass items produced are defective.

21. Winning a Prize A cereal maker places a game piece in each of its cereal boxes. The probability of winning a prize in the game is 1 in 4. Find the probability that you (a) win your first prize with your fourth purchase, (b) win your first prize with your first, second, or third purchase, and (c) do not win a prize with your first four purchases.

22. Precipitation The mean number of days with 0.01 inch or more precipitation per month in Baltimore, Maryland, is about 9.5. Find the probability that in a given month, (a) there are exactly 10 days with 0.01 inch or more precipitation, (b) there are at most 10 days with 0.01 inch or more precipitation, and (c) there are more than 10 days with 0.01 inch or more precipitation. *(Source: National Climatic Data Center)*

SC *In Exercises 23 and 24, use the StatCrunch Poisson calculator to find the indicated probabilities. Then determine if the events are unusual. Explain your reasoning.*

23. Oil Tankers The mean number of oil tankers at a port city is 8 per day. The port has facilities to handle up to 12 oil tankers in a day. Find the probability that on a given day, (a) eight oil tankers will arrive, (b) at most three oil tankers will arrive, and (c) too many oil tankers will arrive.

24. Kidney Transplants The mean number of kidney transplants performed per day in the United States in a recent year was about 45. Find the probability that on a given day, (a) exactly 50 kidney transplants will be performed, (b) at least 65 kidney transplants will be performed, and (c) no more than 40 kidney transplants will be performed. *(Source: U.S. Department of Health and Human Services)*

EXTENDING CONCEPTS

25. Comparing Binomial and Poisson Distributions An automobile manufacturer finds that 1 in every 2500 automobiles produced has a particular manufacturing defect. (a) Use a binomial distribution to find the probability of finding 4 cars with the defect in a random sample of 6000 cars. (b) The Poisson distribution can be used to approximate the binomial distribution for large values of n and small values of p. Repeat (a) using a Poisson distribution and compare the results.

26. Hypergeometric Distribution Binomial experiments require that any sampling be done with replacement because each trial must be independent of the others. The **hypergeometric distribution** also has two outcomes: success and failure. However, the sampling is done without replacement. Given a population of N items having k successes and $N - k$ failures, the probability of selecting a sample of size n that has x successes and $n - x$ failures is given by

$$P(x) = \frac{({}_kC_x)({}_{N-k}C_{n-x})}{{}_NC_n}.$$

In a shipment of 15 microchips, 2 are defective and 13 are not defective. A sample of three microchips is chosen at random. Find the probability that (a) all three microchips are not defective, (b) one microchip is defective and two are not defective, and (c) two microchips are defective and one is not defective.

Geometric Distribution: Mean and Variance *In Exercises 27 and 28, use the fact that the mean of a geometric distribution is* $\mu = 1/p$ *and the variance is* $\sigma^2 = q/p^2$.

27. Daily Lottery A daily number lottery chooses three balls numbered 0 to 9. The probability of winning the lottery is 1/1000. Let x be the number of times you play the lottery before winning the first time. (a) Find the mean, variance, and standard deviation. Interpret the results. (b) How many times would you expect to have to play the lottery before winning? Assume that it costs \$1 to play and winners are paid \$500. Would you expect to make or lose money playing this lottery? Explain.

28. Paycheck Errors A company assumes that 0.5% of the paychecks for a year were calculated incorrectly. The company has 200 employees and examines the payroll records from one month. (a) Find the mean, variance, and standard deviation. Interpret the results. (b) How many employee payroll records would you expect to examine before finding one with an error?

Poisson Distribution: Variance *In Exercises 29 and 30, use the fact that the variance of a Poisson distribution is* $\sigma^2 = \mu$.

29. Golf In a recent year, the mean number of strokes per hole for golfer Phil Mickelson was about 3.9. (a) Find the variance and standard deviation. Interpret the results. (b) How likely is Phil to play an 18-hole round and have more than 72 strokes? *(Source: PGATour.com)*

30. Snowfall The mean snowfall in January in Mount Shasta, California is 29.9 inches. (a) Find the variance and standard deviation. Interpret the results. (b) Find the probability that the snowfall in January in Mount Shasta, California will exceed 3 feet. *(Source: National Climatic Data Center)*

USES AND ABUSES

Uses

There are countless occurrences of binomial probability distributions in business, science, engineering, and many other fields.

For instance, suppose you work for a marketing agency and are in charge of creating a television ad for Brand A toothpaste. The toothpaste manufacturer claims that 40% of toothpaste buyers prefer its brand. To check whether the manufacturer's claim is reasonable, your agency conducts a survey. Of 100 toothpaste buyers selected at random, you find that only 35 (or 35%) prefer Brand A. Could the manufacturer's claim still be true? What if your random sample of 100 found only 25 people (or 25%) who express a preference for Brand A? Would you still be justified in running the advertisement?

Knowing the characteristics of binomial probability distributions will help you answer this type of question. By the time you have completed this course, you will be able make educated decisions about the reasonableness of the manufacturer's claim.

Ethics

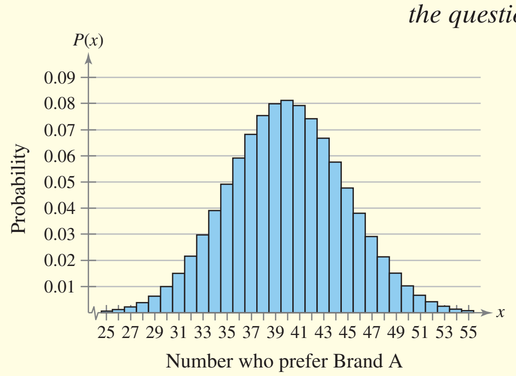
Suppose the toothpaste manufacturer also claims that four out of five dentists recommend Brand A toothpaste. Your agency wants to mention this fact in the television ad, but when determining how the sample of dentists was formed, you find that the dentists were paid to recommend the toothpaste. Including this statement when running the advertisement would be unethical.

Abuses

Interpreting the "Most Likely" Outcome A common misuse of binomial probability distributions is to think that the "most likely" outcome is the outcome that will occur most of the time. For instance, suppose you randomly choose a committee of four from a large population that is 50% women and 50% men. The most likely composition of the committee will be two men and two women. Although this is the most likely outcome, the probability that it will occur is only 0.375. There is a 0.5 chance that the committee will contain one man and three women or three men and one woman. So, if either of these outcomes occurs, you should not assume that the selection was unusual or biased.

EXERCISES

In Exercises 1–4, suppose that the manufacturer's claim is true—40% of toothpaste buyers prefer Brand A toothpaste. Use the graph and technology to answer the questions. Explain your reasoning.



1. ***Interpreting the "Most Likely" Outcome*** In a random sample of 100, what is the most likely outcome? How likely is it?

2. ***Interpreting the "Most Likely" Outcome*** In a random sample of 100, what is the probability that between 35 and 45 people, inclusive, prefer Brand A?

3. Suppose in a random sample of 100, you found 36 who prefer Brand A. Would the manufacturer's claim be believable?

4. Suppose in a random sample of 100, you found 25 who prefer Brand A. Would the manufacturer's claim be believable?

4 CHAPTER SUMMARY

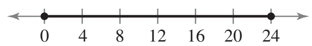
What did you learn?	EXAMPLE(S)	REVIEW EXERCISES
Section 4.1		
■ How to distinguish between discrete random variables and continuous random variables	*1*	*1–6*
■ How to determine if a distribution is a probability distribution	*3–4*	*7–10*
■ How to construct a discrete probability distribution and its graph and find the mean, variance, and standard deviation of a discrete probability distribution	*2, 5, 6*	*11–14*
$\mu = \Sigma xP(x)$ — Mean of a discrete random variable		
$\sigma^2 = \Sigma(x - \mu)^2 P(x)$ — Variance of a discrete random variable		
$\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x - \mu)^2 P(x)}$ — Standard deviation of a discrete random variable		
■ How to find the expected value of a discrete probability distribution	*7*	*15–16*
Section 4.2		
■ How to determine if a probability experiment is a binomial experiment	*1*	*17–20*
■ How to find binomial probabilities using the binomial probability formula, a binomial probability table, and technology	*2, 4–6*	*21–24*
$P(x) = {}_nC_x p^x q^{n-x} = \dfrac{n!}{(n-x)!x!} p^x q^{n-x}$ — Binomial probability formula		
■ How to construct a binomial distribution and its graph and find the mean, variance, and standard deviation of a binomial probability distribution	*3, 7, 8*	*25–28*
$\mu = np$ — Mean of a binomial distribution		
$\sigma^2 = npq$ — Variance of a binomial distribution		
$\sigma = \sqrt{npq}$ — Standard deviation of a binomial distribution		
Section 4.3		
■ How to find probabilities using the geometric distribution	*1*	*29, 30*
$P(x) = pq^{x-1}$ — Probability that the first success will occur on trial number x		
■ How to find probabilities using the Poisson distribution	*2, 3*	*31–33*
$P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$ — Probability of exactly x occurrences in an interval		

4 REVIEW EXERCISES

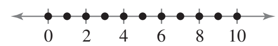
SECTION 4.1

In Exercises 1 and 2, decide whether the graph represents a discrete random variable or a continuous random variable. Explain your reasoning.

1. The number of hours spent sleeping each day



2. The number of fish caught during a fishing tournament



In Exercises 3–6, decide whether the random variable x is discrete or continuous.

3. Let x represent the number of pumps in use at a gas station.

4. Let x represent the weight of a truck at a weigh station.

5. Let x represent the amount of carbon dioxide emitted from a car's tailpipe each day.

6. Let x represent the number of people that activate a metal detector at an airport each hour.

In Exercises 7–10, decide whether the distribution is a probability distribution. If it is not, identify the property that is not satisfied.

7. The daily limit for catching bass at a lake is four. The random variable x represents the number of fish caught in a day.

x	0	1	2	3	4
$P(x)$	0.36	0.23	0.08	0.14	0.29

8. The random variable x represents the number of tickets a police officer writes out each shift.

x	0	1	2	3	4	5
$P(x)$	0.09	0.23	0.29	0.16	0.21	0.02

9. A greeting card shop keeps records of customers' buying habits. The random variable x represents the number of cards sold to an individual customer in a shopping visit.

x	1	2	3	4	5	6	7
$P(x)$	0.68	0.14	0.08	0.05	0.02	0.02	0.01

10. The random variable x represents the number of classes in which a student is enrolled in a given semester at a university.

x	1	2	3	4	5	6	7	8
$P(x)$	$\frac{1}{80}$	$\frac{2}{75}$	$\frac{1}{10}$	$\frac{12}{25}$	$\frac{27}{20}$	$\frac{1}{5}$	$\frac{2}{25}$	$\frac{1}{120}$

In Exercises 11–14,

(a) use the frequency distribution table to construct a probability distribution,

(b) graph the probability distribution using a histogram and describe its shape,

(c) find the mean, variance, and standard deviation of the probability distribution, and

(d) interpret the results in the context of the real-life situation.

11. The number of pages in a section from 10 statistics texts

Pages	Sections
2	3
3	12
4	72
5	115
6	169
7	120
8	83
9	48
10	22
11	6

12. The number of hits per game played by a baseball player during a recent season

Hits	Games
0	29
1	62
2	33
3	12
4	3
5	1

13. The distribution of the number of cellular phones per household in a small town is given.

Cellphones	Families
0	5
1	35
2	68
3	73
4	42
5	19
6	8

14. A television station sells advertising in 15-, 30-, 60-, 90-, and 120-second blocks. The distribution of sales for one 24-hour day is given.

Length (in seconds)	Number
15	76
30	445
60	30
90	3
120	12

In Exercises 15 and 16, find the expected value of the random variable.

15. A person has shares of eight different stocks. The random variable x represents the number of stocks showing a loss on a selected day.

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.02	0.11	0.18	0.32	0.15	0.09	0.05	0.05	0.03

16. A local pub has a chicken wing special on Tuesdays. The pub owners purchase wings in cases of 300. The random variable x represents the number of cases used during the special.

x	1	2	3	4
$P(x)$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{18}$

SECTION 4.2

In Exercises 17 and 18, use the following information.

A probability experiment has n independent trials. Each trial has three possible outcomes: A, B, and C. For each trial, $P(A) = 0.30$, $P(B) = 0.50$, and $P(C) = 0.20$. There are 20 trials.

17. Can a binomial experiment be used to find the probability of 6 outcomes of A, 10 outcomes of B, and 4 outcomes of C? Explain your reasoning.

18. Can a binomial experiment be used to find the probability of 4 outcomes of C and 16 outcomes that are *not* C? Explain your reasoning. What is the probability of success for each trial?

In Exercises 19 and 20, decide whether the experiment is a binomial experiment. If it is not, identify the property that is not satisfied. If it is, list the values of n, p, and q and the values that x can assume.

19. Bags of plain M&M's contain 24% blue candies. One candy is selected from each of 12 bags. The random variable represents the number of blue candies selected. *(Source: Mars, Incorporated)*

20. A fair coin is tossed repeatedly until 15 heads are obtained. The random variable x counts the number of tosses.

In Exercises 21–24, find the indicated probabilities.

21. One in four adults is currently on a diet. You randomly select eight adults and ask them if they are currently on a diet. Find the probability that the number who say they are currently on a diet is (a) exactly three, (b) at least three, and (c) more than three. *(Source: Wirthlin Worldwide)*

22. One in four people in the United States owns individual stocks. You randomly select 12 people and ask them if they own individual stocks. Find the probability that the number who say they own individual stocks is (a) exactly two, (b) at least two, and (c) more than two. *(Source: Pew Research Center)*

23. Forty-three percent of businesses in the United States require a doctor's note when an employee takes sick time. You randomly select nine businesses and ask each if it requires a doctor's note when an employee takes sick time. Find the probability that the number who say they require a doctor's note is (a) exactly five, (b) at least five, and (c) more than five. *(Source: Harvard School of Public Health)*

24. In a typical day, 31% of people in the United States with Internet access go online to get news. You randomly select five people in the United States with Internet access and ask them if they go online to get news. Find the probability that the number who say they go online to get news is (a) exactly two, (b) at least two, and (c) more than two. *(Source: Pew Research Center)*

In Exercises 25–28,

(a) construct a binomial distribution,

(b) graph the binomial distribution using a histogram and describe its shape,

(c) find the mean, variance, and standard deviation of the binomial distribution and interpret the results in the context of the real-life situation, and

(d) determine the values of the random variable x that you would consider unusual.

25. Thirty-four percent of women in the United States say their spouses never help with household chores. You randomly select five U.S. women and ask if their spouses help with household chores. *(Source: Boston Consulting Group)*

26. Sixty-eight percent of families say that their children have an influence on their vacation destinations. You randomly select six families and ask if their children have an influence on their vacation destinations. *(Source: YPB&R)*

27. In a recent year, forty percent of trucks sold by a company had diesel engines. You randomly select four trucks sold by the company and check if they have diesel engines.

28. Sixty-three percent of U.S. mothers with school-age children choose fast food as a dining option for their families one to three times a week. You randomly select five U.S. mothers with school-age children and ask if they choose fast food as a dining option for their families one to three times a week. *(Adapted from Market Day)*

SECTION 4.3

In Exercises 29–32, find the indicated probabilities using the geometric distribution or the Poisson distribution. Then determine if the events are unusual. If convenient, use a Poisson probability table or technology to find the probabilities.

29. Twenty-two percent of former smokers say they tried to quit four or more times before they were habit-free. You randomly select 10 former smokers. Find the probability that the first person who tried to quit four or more times is (a) the third person selected, (b) the fourth or fifth person selected, and (c) not one of the first seven people selected. *(Source: Porter Novelli Health Styles)*

30. In a recent season, hockey player Sidney Crosby scored 33 goals in 77 games he played. Assume that his goal production stayed at that level the following season. What is the probability that he would get his first goal

(a) in the first game of the season?

(b) in the second game of the season?

(c) in the first or second game of the season?

(d) within the first three games of the season? *(Source: ESPN)*

31. During a 69-year period, tornadoes killed 6755 people in the United States. Assume this rate holds true today and is constant throughout the year. Find the probability that tomorrow

(a) no one in the U. S. is killed by a tornado,

(b) one person in the U.S. is killed by a tornado,

(c) at most two people in the U.S. are killed by a tornado, and

(d) more than one person in the U.S. is killed by a tornado. *(Source: National Weather Service)*

32. It is estimated that sharks kill 10 people each year worldwide. Find the probability that at least 3 people are killed by sharks this year

(a) assuming that this rate is true,

(b) if the rate is actually 5 people a year, and

(c) if the rate is actually 15 people a year. *(Source: International Shark Attack File)*

33. In Exercise 32, describe what happens to the probability of at least three people being killed by sharks this year as the rate increases and decreases.

4 CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

1. Decide if the random variable x is discrete or continuous. Explain your reasoning.

(a) Let x represent the number of lightning strikes that occur in Wyoming during the month of June.

(b) Let x represent the amount of fuel (in gallons) used by the Space Shuttle during takeoff.

2. The table lists the number of U.S. mainland hurricane strikes (from 1851 to 2008) for various intensities according to the Saffir-Simpson Hurricane Scale. *(Source: National Oceanic and Atmospheric Administration)*

Intensity	Number of hurricanes
1	114
2	74
3	76
4	18
5	3

TABLE FOR EXERCISE 2

(a) Construct a probability distribution of the data.

(b) Graph the discrete probability distribution using a probability histogram. Then describe its shape.

(c) Find the mean, variance, and standard deviation of the probability distribution and interpret the results.

(d) Find the probability that a hurricane selected at random for further study has an intensity of at least four.

3. The success rate of corneal transplant surgery is 85%. The surgery is performed on six patients. *(Adapted from St. Luke's Cataract & Laser Institute)*

(a) Construct a binomial distribution.

(b) Graph the binomial distribution using a probability histogram. Then describe its shape.

(c) Find the mean, variance, and standard deviation of the probability distribution and interpret the results.

(d) Find the probability that the surgery is successful for exactly three patients. Is this an unusual event? Explain.

(e) Find the probability that the surgery is successful for fewer than four patients. Is this an unusual event? Explain.

4. A newspaper finds that the mean number of typographical errors per page is five. Find the probability that

(a) exactly five typographical errors will be found on a page,

(b) fewer than five typographical errors will be found on a page, and

(c) no typographical errors will be found on a page.

In Exercises 5 and 6, use the following information. Basketball player Dwight Howard makes a free throw shot about 60.2% of the time. *(Source: ESPN)*

5. Find the probability that the first free throw shot Dwight makes is the fourth shot. Is this an unusual event? Explain.

6. Find the probability that the first free throw shot Dwight makes is the second or third shot. Is this an unusual event? Explain.

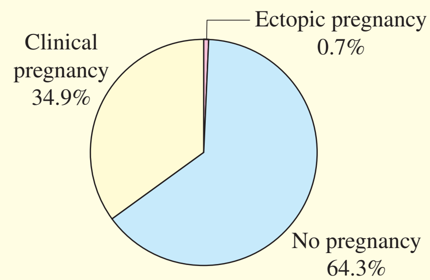
PUTTING IT ALL TOGETHER

Real Statistics — Real Decisions

The Centers for Disease Control and Prevention (CDC) is required by law to publish a report on assisted reproductive technologies (ART). ART includes all fertility treatments in which both the egg and the sperm are used. These procedures generally involve removing eggs from a woman's ovaries, combining them with sperm in the laboratory, and returning them to the woman's body or giving them to another woman.

You are helping to prepare the CDC report and select at random 10 ART cycles for a special review. None of the cycles resulted in a clinical pregnancy. Your manager feels it is impossible to select at random 10 ART cycles that did not result in a clinical pregnancy. Use the information provided at the right and your knowledge of statistics to determine if your manager is correct.

Results of ART Cycles Using Fresh Nondonor Eggs or Embryos

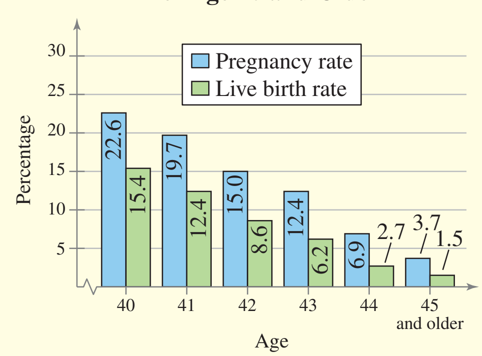


(Source: Centers for Disease Control and Prevention)

EXERCISES

1. ***How Would You Do It?***
   (a) How would you determine if your manager's view is correct, that it is impossible to select at random 10 ART cycles that did not result in a clinical pregnancy?
   (b) What probability distribution do you think best describes the situation? Do you think the distribution of the number of clinical pregnancies is discrete or continuous? Why?
2. ***Answering the Question***
   Write an explanation that answers the question, "Is it possible to select at random 10 ART cycles that did not result in a clinical pregnancy?" Include in your explanation the appropriate probability distribution and your calculation of the probability of no clinical pregnancies in 10 ART cycles.
3. ***Suspicious Samples?***
   Which of the following samples would you consider suspicious if someone told you that the sample was selected at random? Would you believe that the samples were selected at random? Why or why not?
   (a) Selecting at random 10 ART cycles among women of age 40, eight of which resulted in clinical pregnancies.
   (b) Selecting at random 10 ART cycles among women of age 41, none of which resulted in clinical pregnancies.

Pregnancy and Live Birth Rates for ART Cycles Among Women of Age 40 and Older



(Source: Centers for Disease Control and Prevention)

TECHNOLOGY

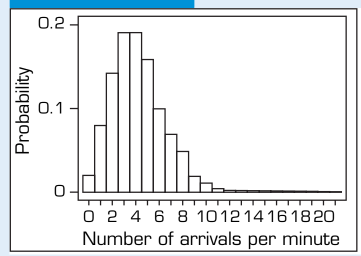
MINITAB | EXCEL | TI-83/84 PLUS

USING POISSON DISTRIBUTIONS AS QUEUING MODELS

Queuing means waiting in line to be served. There are many examples of queuing in everyday life: waiting at a traffic light, waiting in line at a grocery checkout counter, waiting for an elevator, holding for a telephone call, and so on.

Poisson distributions are used to model and predict the number of people (calls, computer programs, vehicles) arriving at the line. In the following exercises, you are asked to use Poisson distributions to analyze the queues at a grocery store checkout counter.

MINITAB



EXERCISES

In Exercises 1–7, consider a grocery store that can process a total of four customers at its checkout counters each minute.

1. Suppose that the mean number of customers who arrive at the checkout counters each minute is 4. Create a Poisson distribution with $\mu = 4$ for $x = 0$ to 20. Compare your results with the histogram shown at the upper right.

2. MINITAB was used to generate 20 random numbers with a Poisson distribution for $\mu = 4$. Let the random number represent the number of arrivals at the checkout counter each minute for 20 minutes.

3	3	3	3	5	5	6	7	3	6
3	5	6	3	4	6	2	2	4	1

During each of the first four minutes, only three customers arrived. These customers could all be processed, so there were no customers waiting after four minutes.

(a) How many customers were waiting after 5 minutes? 6 minutes? 7 minutes? 8 minutes?

(b) Create a table that shows the number of customers waiting at the end of 1 through 20 minutes.

3. Generate a list of 20 random numbers with a Poisson distribution for $\mu = 4$. Create a table that shows the number of customers waiting at the end of 1 through 20 minutes.

4. Suppose that the mean increases to 5 arrivals per minute. You can still process only four per minute. How many would you expect to be waiting in line after 20 minutes?

5. Simulate the setting in Exercise 4. Do this by generating a list of 20 random numbers with a Poisson distribution for $\mu = 5$. Then create a table that shows the number of customers waiting at the end of 20 minutes.

6. Suppose that the mean number of arrivals per minute is 5. What is the probability that 10 customers will arrive during the first minute?

7. Suppose that the mean number of arrivals per minute is 4.

(a) What is the probability that three, four, or five customers will arrive during the third minute?

(b) What is the probability that more than four customers will arrive during the first minute?

(c) What is the probability that more than four customers will arrive during each of the first four minutes?

Extended solutions are given in the *Technology Supplement.*
Technical instruction is provided for MINITAB, Excel, and the TI-83/84 Plus.