7

CHAPTER

HYPOTHESIS TESTING WITH ONE SAMPLE

Computer software is protected by federal copyright laws. Each year, software companies lose billions of dollars because of pirated software. Federal criminal penalties for software piracy can include fines of up to $250,000 and jail terms of up to five years.

WHERE YOU'VE BEEN

In Chapter 6, you began your study of inferential statistics. There, you learned how to form a confidence interval estimate about a population parameter, such as the proportion of people in the United States who agree with a certain statement. For instance, in a nationwide poll conducted by *Harris Interactive* on behalf of the Business Software Alliance (BSA), U.S. students ages 8 to 18 years were asked several questions about their attitudes toward copyright law and Internet behavior. Here are some of the results.

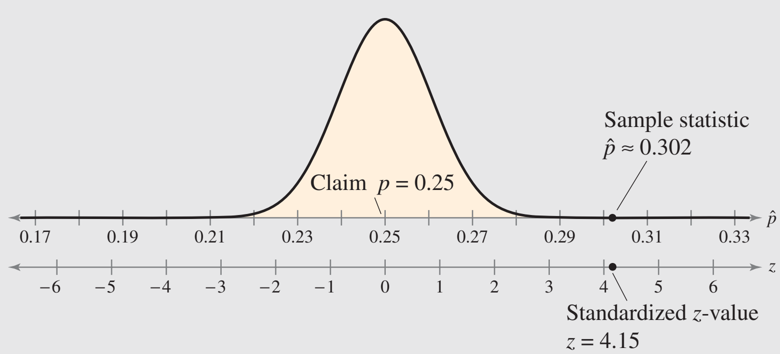
Survey Question	Number Surveyed	Number Who Said Yes
Have you ever downloaded music from the Internet without paying for it?	1196	361
Have you ever downloaded movies from the Internet without paying for them?	1196	95
Have you ever downloaded software from the Internet without paying for it?	1196	133

WHERE YOU'RE GOING

In this chapter, you will continue your study of inferential statistics. But now, instead of making an estimate about a population parameter, you will learn how to test a claim about a parameter.

For instance, suppose that you work for *Harris Interactive* and are asked to test a claim that the proportion of U.S. students ages 8 to 18 who download music without paying for it is $p = 0.25$. To test the claim, you take a random sample of $n = 1196$ students and find that 361 of them download music without paying for it. Your sample statistic is $\hat{p} \approx 0.302$.

Is your sample statistic different enough from the claim ($p = 0.25$) to decide that the claim is false? The answer lies in the sampling distribution of sample proportions taken from a population in which $p = 0.25$. The graph below shows that your sample statistic is more than 4 standard errors from the claimed value. If the claim is true, the probability of the sample statistic being 4 standard errors or more from the claimed value is extremely small. Something is wrong! If your sample was truly random, then you can conclude that the actual proportion of the student population is not 0.25. In other words, you tested the original claim (hypothesis), and you decided to reject it.



Sampling Distribution

7.1 Introduction to Hypothesis Testing

WHAT YOU SHOULD LEARN

- A practical introduction to hypothesis tests
- How to state a null hypothesis and an alternative hypothesis
- How to identify type I and type II errors and interpret the level of significance
- How to know whether to use a one-tailed or two-tailed statistical test and find a *P*-value
- How to make and interpret a decision based on the results of a statistical test
- How to write a claim for a hypothesis test

Hypothesis Tests ▸ Stating a Hypothesis ▸ Types of Errors and Level of Significance ▸ Statistical Tests and *P*-Values ▸ Making a Decision and Interpreting the Decision ▸ Strategies for Hypothesis Testing

▸ HYPOTHESIS TESTS

Throughout the remainder of this course, you will study an important technique in inferential statistics called hypothesis testing. A **hypothesis test** is a process that uses sample statistics to test a claim about the value of a population parameter. Researchers in fields such as medicine, psychology, and business rely on hypothesis testing to make informed decisions about new medicines, treatments, and marketing strategies.

For instance, suppose an automobile manufacturer advertises that its new hybrid car has a mean gas mileage of 50 miles per gallon. If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

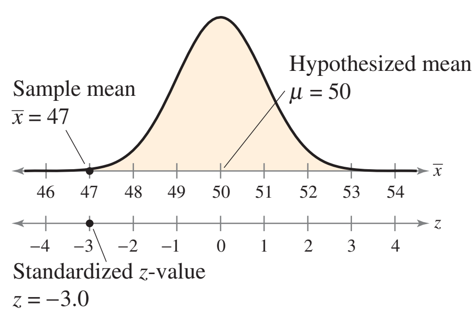
Obviously, you cannot test *all* the vehicles, but you can still make a reasonable decision about the mean gas mileage by taking a random sample from the population of vehicles and measuring the mileage of each. If the sample mean differs enough from the advertisement's mean, you can decide that the advertisement is wrong.

For instance, to test that the mean gas mileage of all hybrid vehicles of this type is $\mu = 50$ miles per gallon, you could take a random sample of $n = 30$ vehicles and measure the mileage of each. Suppose you obtain a sample mean of $\bar{x} = 47$ miles per gallon with a sample standard deviation of $s = 5.5$ miles per gallon. Does this indicate that the manufacturer's advertisement is false?

To decide, you do something unusual—*you assume the advertisement is correct!* That is, you assume that $\mu = 50$. Then, you examine the sampling distribution of sample means (with $n = 30$) taken from a population in which $\mu = 50$ and $\sigma = 5.5$. From the Central Limit Theorem, you know this sampling distribution is normal with a mean of 50 and standard error of

$$\frac{5.5}{\sqrt{30}} \approx 1.$$

Sampling Distribution of $\bar{x}$



In the graph at the right, notice that your sample mean of $\bar{x} = 47$ miles per gallon is highly unlikely—it is about 3 standard errors from the claimed mean! Using the techniques you studied in Chapter 5, you can determine that if the advertisement is true, the probability of obtaining a sample mean of 47 or less is about 0.0013. This is an unusual event! Your assumption that the company's advertisement is correct has led you to an improbable result. So, either you had a very unusual sample, or the advertisement is probably false. The logical conclusion is that the advertisement is probably false.

INSIGHT

As you study this chapter, don't get confused regarding concepts of certainty and importance. For instance, even if you were very certain that the mean gas mileage of a type of hybrid vehicle is not 50 miles per gallon, the actual mean mileage might be very close to this value and the difference might not be important.

STATING A HYPOTHESIS

A statement about a population parameter is called a **statistical hypothesis.** To test a population parameter, you should carefully state a pair of hypotheses—one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true. Either hypothesis—the *null hypothesis* or the *alternative hypothesis*—may represent the original claim.

INSIGHT

The term *null hypothesis* was introduced by Ronald Fisher (see page 33). If the statement in the null hypothesis is not true, then the alternative hypothesis must be true.

DEFINITION

1. A **null hypothesis H_0** is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis H_a** is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

H_0 is read as "H sub-zero" or "H naught" and H_a is read as "H sub-a."

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement. Then, write its complement. For instance, if the claim value is k and the population parameter is μ, then some possible pairs of null and alternative hypotheses are

$$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases} \qquad \begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases} \qquad \begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}.$$

Regardless of which of the three pairs of hypotheses you use, you always assume $\mu = k$ and examine the sampling distribution on the basis of this assumption. Within this sampling distribution, you will determine whether or not a sample statistic is unusual.

The following table shows the relationship between possible verbal statements about the parameter μ and the corresponding null and alternative hypotheses. Similar statements can be made to test other population parameters, such as p, σ, or σ^2.

Verbal Statement H_0 *The mean is . . .*	**Mathematical Statements**	**Verbal Statement H_a** *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.*
. . . equal to k. *. . . k.* *. . . exactly k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.*

PICTURING THE WORLD

A sample of 25 randomly selected patients with early-stage high blood pressure underwent a special chiropractic adjustment to help lower their blood pressure. After eight weeks, the mean drop in the patients' systolic blood pressure was 14 millimeters of mercury. So, it is claimed that the mean drop in systolic blood pressure of all patients who undergo this special chiropractic adjustment is 14 millimeters of mercury. (Adapted from Journal of Human Hypertension)

Determine a null hypothesis and alternative hypothesis for this claim.

EXAMPLE 1

Stating the Null and Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses, and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

Solution

1. The claim "the proportion . . . is 61%" can be written as $p = 0.61$. Its complement is $p \neq 0.61$. Because $p = 0.61$ contains the statement of equality, it becomes the null hypothesis. In this case, the null hypothesis represents the claim.

   H_0: $p = 0.61$ (Claim)

   H_a: $p \neq 0.61$

2. The claim "the mean . . . is less than 15 minutes" can be written as $\mu < 15$. Its complement is $\mu \geq 15$. Because $\mu \geq 15$ contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim.

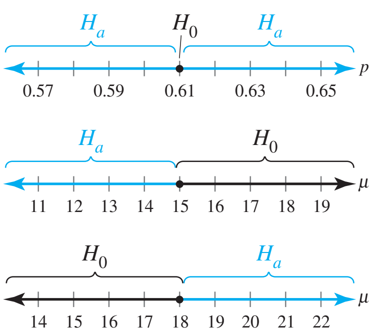
   H_0: $\mu \geq 15$ minutes

   H_a: $\mu < 15$ minutes (Claim)

3. The claim "the mean . . . is more than 18 years" can be written as $\mu > 18$. Its complement is $\mu \leq 18$. Because $\mu \leq 18$ contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim.

   H_0: $\mu \leq 18$ years

   H_a: $\mu > 18$ years (Claim)



In each of these graphs, notice that each point on the number line is in H_0 or H_a, but no point is in both.

Try It Yourself 1

Write the claim as a mathematical sentence. State the null and alternative hypotheses, and identify which represents the claim.

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.
3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

a. Identify the *verbal claim* and write it as a *mathematical statement*.
b. Write the *complement* of the claim.
c. Identify the *null* and *alternative hypotheses* and determine which one represents the claim.

Answer: Page A40

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

No matter which hypothesis represents the claim, you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis or
2. fail to reject the null hypothesis.

Because your decision is based on a sample rather than the entire population, there is always the possibility you will make the wrong decision.

For instance, suppose you claim that a certain coin is not fair. To test your claim, you flip the coin 100 times and get 49 heads and 51 tails. You would probably agree that you do not have enough evidence to support your claim. Even so, it is possible that the coin is actually not fair and you had an unusual sample.

But what if you flip the coin 100 times and get 21 heads and 79 tails? It would be a rare occurrence to get only 21 heads out of 100 tosses with a fair coin. So, you probably have enough evidence to support your claim that the coin is not fair. However, you can't be 100% sure. It is possible that the coin is fair and you had an unusual sample.

If p represents the proportion of heads, the claim that "the coin is not fair" can be written as the mathematical statement $p \neq 0.5$. Its complement, "the coin is fair," is written as $p = 0.5$. So, your null hypothesis and alternative hypothesis are

$$H_0\colon p = 0.5$$

and

$$H_a\colon p \neq 0.5. \quad \text{(Claim)}$$

Remember, the only way to be absolutely certain of whether H_0 is true or false is to test the entire population. Because your decision—to reject H_0 or to fail to reject H_0—is based on a sample, you must accept the fact that your decision might be incorrect. You might reject a null hypothesis when it is actually true. Or, you might fail to reject a null hypothesis when it is actually false.

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

The following table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	**H_0 is true.**	**H_0 is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

	Truth About Defendant	
Verdict	**Innocent**	**Guilty**
Not guilty	Justice	Type II error
Guilty	Type I error	Justice

Hypothesis testing is sometimes compared to the legal system used in the United States. Under this system, the following steps are used.

1. A carefully worded accusation is written.
2. The defendant is assumed innocent (H_0) until proven guilty. The burden of proof lies with the prosecution. If the evidence is not strong enough, there is no conviction. A "not guilty" verdict does not prove that a defendant is innocent.
3. The evidence needs to be conclusive beyond a reasonable doubt. The system assumes that more harm is done by convicting the innocent (type I error) than by not convicting the guilty (type II error).

EXAMPLE 2

▸ Identifying Type I and Type II Errors

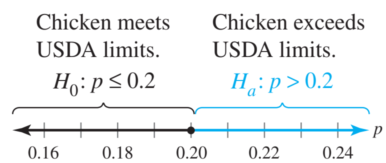
The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which is more serious? *(Source: U.S. Department of Agriculture)*

▸ Solution

Let p represent the proportion of the chicken that is contaminated. The meat inspector's claim is "more than 20% is contaminated." You can write the null and alternative hypotheses as follows.

H_0: $p \leq 0.2$ The proportion is less than or equal to 20%.

H_a: $p > 0.2$ (Claim) The proportion is greater than 20%.



A type I error will occur if the actual proportion of contaminated chicken is less than or equal to 0.2, but you reject H_0. A type II error will occur if the actual proportion of contaminated chicken is greater than 0.2, but you do not reject H_0. With a type I error, you might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits. With a type II error, you could be allowing chicken that exceeded the USDA contamination limit to be sold to consumers. A type II error is more serious because it could result in sickness or even death.

▸ Try It Yourself 2

A company specializing in parachute assembly states that its main parachute failure rate is not more than 1%. You perform a hypothesis test to determine whether the company's claim is false. When will a type I or type II error occur? Which is more serious?

a. State the *null* and *alternative hypotheses*.
b. Write the possible *type I* and *type II* errors.
c. *Determine* which error is more serious.

Answer: Page A40

You will reject the null hypothesis when the sample statistic from the sampling distribution is unusual. You have already identified unusual events to be those that occur with a probability of 0.05 or less. When statistical tests are used, an unusual event is sometimes required to have a probability of 0.10 or less, 0.05 or less, or 0.01 or less. Because there is variation from sample to sample, there is always a possibility that you will reject a null hypothesis when it is actually true. In other words, although the null hypothesis is true, your sample statistic is determined to be an unusual event in the sampling distribution. You can decrease the probability of this happening by lowering the *level of significance*.

INSIGHT

When you decrease α (the maximum allowable probability of making a type I error), you are likely to be increasing β. The value $1 - \beta$ is called the **power of the test.** It represents the probability of rejecting the null hypothesis when it is false. The value of the power is difficult (and sometimes impossible) to find in most cases.

DEFINITION

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

The probability of a type II error is denoted by β, the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small. Three commonly used levels of significance are $\alpha = 0.10$, $\alpha = 0.05$, and $\alpha = 0.01$.

STATISTICAL TESTS AND *P*-VALUES

After stating the null and alternative hypotheses and specifying the level of significance, the next step in a hypothesis test is to obtain a random sample from the population and calculate sample statistics such as the mean and the standard deviation. The statistic that is compared with the parameter in the null hypothesis is called the **test statistic.** The type of test used and the sampling distribution are based on the test statistic.

In this chapter, you will learn about several one-sample statistical tests. The following table shows the relationships between population parameters and their corresponding test statistics and standardized test statistics.

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, $n \geq 30$), t (Section 7.3, $n < 30$)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

One way to decide whether to reject the null hypothesis is to determine whether the probability of obtaining the standardized test statistic (or one that is more extreme) is less than the level of significance.

DEFINITION

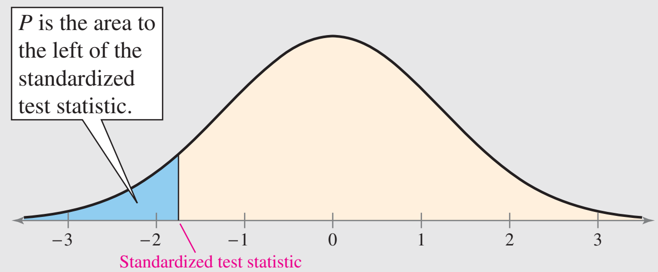
If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

The P-value of a hypothesis test depends on the nature of the test. There are three types of hypothesis tests—left-tailed, right-tailed, and two-tailed. The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

DEFINITION

1. If the alternative hypothesis H_a contains the less-than inequality symbol $(<)$, the hypothesis test is a **left-tailed test.**

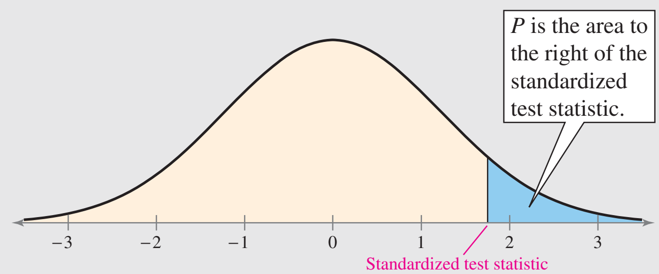
H_0: $\mu \geq k$
H_a: $\mu < k$



Left-Tailed Test

2. If the alternative hypothesis H_a contains the greater-than inequality symbol $(>)$, the hypothesis test is a **right-tailed test.**

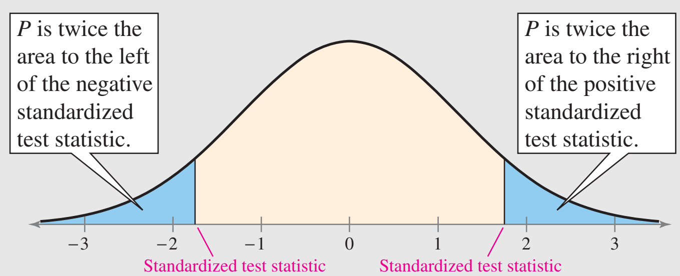
H_0: $\mu \leq k$
H_a: $\mu > k$



Right-Tailed Test

3. If the alternative hypothesis H_a contains the not-equal-to symbol $(\neq)$, the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.

H_0: $\mu = k$
H_a: $\mu \neq k$



Two-Tailed Test

STUDY TIP

The third type of test is called a two-tailed test because evidence that would support the alternative hypothesis could lie in either tail of the sampling distribution.

The smaller the P-value of the test, the more evidence there is to reject the null hypothesis. A very small P-value indicates an unusual event. Remember, however, that even a very low P-value does not constitute proof that the null hypothesis is false, only that it is probably false.

EXAMPLE 3

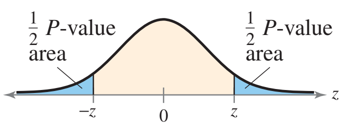
Identifying the Nature of a Hypothesis Test

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the P-value.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

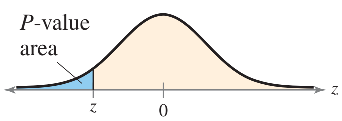
Solution

	In Symbols	***In Words***
1.	H_0: $p = 0.61$	The proportion of students who are involved in at least one extracurricular activity is 61%.
	H_a: $p \neq 0.61$	The proportion of students who are involved in at least one extracurricular activity is not 61%.



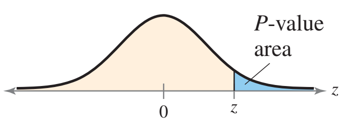
Because H_a contains the $\neq$ symbol, the test is a two-tailed hypothesis test. The graph of the normal sampling distribution at the left shows the shaded area for the P-value.

2.	H_0: $\mu \geq 15$ min	The mean time for an oil change is greater than or equal to 15 minutes.
	H_a: $\mu < 15$ min	The mean time for an oil change is less than 15 minutes.



Because H_a contains the $<$ symbol, the test is a left-tailed hypothesis test. The graph of the normal sampling distribution at the left shows the shaded area for the P-value.

3.	H_0: $\mu \leq 18$ yr	The mean life of the furnaces is less than or equal to 18 years.
	H_a: $\mu > 18$ yr	The mean life of the furnaces is more than 18 years.



Because H_a contains the $>$ symbol, the test is a right-tailed hypothesis test. The graph of the normal sampling distribution at the left shows the shaded area for the P-value.

Try It Yourself 3

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the P-value.

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.
3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

a. *Write* H_0 and H_a in words and in symbols.
b. Determine whether the test is *left-tailed*, *right-tailed*, or *two-tailed*.
c. *Sketch* the sampling distribution and *shade* the area for the P-value.

Answer: Page A40

MAKING A DECISION AND INTERPRETING THE DECISION

To conclude a hypothesis test, you make a decision and interpret that decision. There are only two possible outcomes to a hypothesis test: (1) reject the null hypothesis and (2) fail to reject the null hypothesis.

INSIGHT

In this chapter, you will learn that there are two basic types of **decision rules** for deciding whether to reject H_0 or fail to reject H_0. The decision rule described on this page is based on *P*-values. The second basic type of decision rule is based on rejection regions. When the standardized test statistic falls in the rejection region, the observed probability (*P*-value) of a type I error is less than α. You will learn more about rejection regions in the next section.

DECISION RULE BASED ON *P*-VALUE

To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.

2. If $P > \alpha$, then fail to reject H_0.

Failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis. If you want to support a claim, state it so that it becomes the alternative hypothesis. If you want to reject a claim, state it so that it becomes the null hypothesis. The following table will help you interpret your decision.

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

EXAMPLE 4

Interpreting a Decision

You perform a hypothesis test for each of the following claims. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. H_0 (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

2. H_a (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

Solution

1. The claim is represented by H_0. If you reject H_0, then you should conclude "there is enough evidence to reject the school's claim that the proportion of students who are involved in at least one extracurricular activity is 61%." If you fail to reject H_0, then you should conclude "there is not enough evidence to reject the school's claim that the proportion of students who are involved in at least one extracurricular activity is 61%."

2. The claim is represented by H_a, so the null hypothesis is "the mean time for an oil change is greater than or equal to 15 minutes." If you reject H_0, then you should conclude "there is enough evidence to support the dealership's claim that the mean time for an oil change is less than 15 minutes." If you fail to reject H_0, then you should conclude "there is not enough evidence to support the dealership's claim that the mean time for an oil change is less than 15 minutes."

Try It Yourself 4

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

H_a (Claim): A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

a. Interpret your decision if you *reject* the null hypothesis.
b. Interpret your decision if you *fail to reject* the null hypothesis.

Answer: Page A41

The general steps for a hypothesis test using *P*-values are summarized below.

STUDY TIP

When performing a hypothesis test, you should always state the null and alternative hypotheses before collecting data. You should not collect the data first and then create a hypothesis based on something unusual in the data.

STEPS FOR HYPOTHESIS TESTING

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

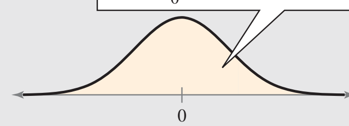
H_0: **?** H_a: **?**

2. Specify the level of significance.

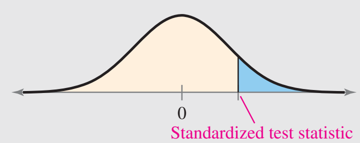
$\alpha =$ **?**

3. Determine the standardized sampling distribution and sketch its graph.

This sampling distribution is based on the assumption that H_0 is true.



4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.



5. Find the *P*-value.
6. Use the following decision rule.

Is the *P*-value less than or equal to the level of significance?

No → Fail to reject H_0.

Yes → Reject H_0.

7. Write a statement to interpret the decision in the context of the original claim.

In the steps above, the graphs show a right-tailed test. However, the same basic steps also apply to left-tailed and two-tailed tests.

STRATEGIES FOR HYPOTHESIS TESTING

In a courtroom, the strategy used by an attorney depends on whether the attorney is representing the defense or the prosecution. In a similar way, the strategy that you will use in hypothesis testing should depend on whether you are trying to support or reject a claim. Remember that you cannot use a hypothesis test to support your claim if your claim is the null hypothesis. So, as a researcher, if you want a conclusion that supports your claim, word your claim so it is the alternative hypothesis. If you want to reject a claim, word it so it is the null hypothesis.

EXAMPLE 5

Writing the Hypotheses

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours. How would you write the null and alternative hypotheses if (1) you are on the research team and want to support the claim? (2) you are on an opposing team and want to reject the claim?

Solution

1. To answer the question, first think about the context of the claim. Because you want to support this claim, make the alternative hypothesis state that the mean recovery time for patients is less than 96 hours. So, H_a: $\mu < 96$ hours. Its complement, $\mu \geq 96$ hours, would be the null hypothesis.

H_0: $\mu \geq 96$

H_a: $\mu < 96$ (Claim)

2. First think about the context of the claim. As an opposing researcher, you do not want the recovery time to be less than 96 hours. Because you want to reject this claim, make it the null hypothesis. So, H_0: $\mu \leq 96$ hours. Its complement, $\mu > 96$ hours, would be the alternative hypothesis.

H_0: $\mu \leq 96$ (Claim)

H_a: $\mu > 96$

Try It Yourself 5

1. You represent a chemical company that is being sued for paint damage to automobiles. You want to support the claim that the mean repair cost per automobile is less than $650. How would you write the null and alternative hypotheses?

2. You are on a research team that is investigating the mean temperature of adult humans. The commonly accepted claim is that the mean temperature is about 98.6°F. You want to show that this claim is false. How would you write the null and alternative hypotheses?

a. *Determine* whether you want to support or reject the claim.

b. Write the *null* and *alternative hypotheses*.

Answer: Page A41

7.1 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. What are the two types of hypotheses used in a hypothesis test? How are they related?

2. Describe the two types of error possible in a hypothesis test decision.

3. What are the two decisions that you can make from performing a hypothesis test?

4. Does failing to reject the null hypothesis mean that the null hypothesis is true? Explain.

True or False? *In Exercises 5–10, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

5. In a hypothesis test, you assume the alternative hypothesis is true.

6. A statistical hypothesis is a statement about a sample.

7. If you decide to reject the null hypothesis, you can support the alternative hypothesis.

8. The level of significance is the maximum probability you allow for rejecting a null hypothesis when it is actually true.

9. A large P-value in a test will favor rejection of the null hypothesis.

10. If you want to support a claim, write it as your null hypothesis.

Stating Hypotheses *In Exercises 11–16, use the given statement to represent a claim. Write its complement and state which is H_0 and which is H_a.*

11. $\mu \le 645$

12. $\mu < 128$

13. $\sigma \ne 5$

14. $\sigma^2 \ge 1.2$

15. $p < 0.45$

16. $p = 0.21$

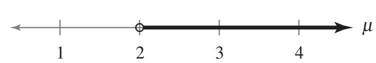
Graphical Analysis *In Exercises 17–20, match the alternative hypothesis with its graph. Then state the null hypothesis and sketch its graph.*

17. $H_a: \mu > 3$

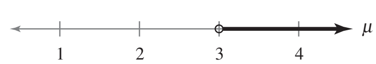
18. $H_a: \mu < 3$

19. $H_a: \mu \ne 3$

20. $H_a: \mu > 2$

(a) 

(b) 

(c) 

(d) 

Identifying Tests *In Exercises 21–24, determine whether the hypothesis test with the given null and alternative hypotheses is left-tailed, right-tailed, or two-tailed.*

21. $H_0: \mu \le 8.0$
$H_a: \mu > 8.0$

22. $H_0: \sigma \ge 5.2$
$H_a: \sigma < 5.2$

23. $H_0: \sigma^2 = 142$
$H_a: \sigma^2 \ne 142$

24. $H_0: p = 0.25$
$H_a: p \ne 0.25$

USING AND INTERPRETING CONCEPTS

Stating the Hypotheses *In Exercises 25–30, write the claim as a mathematical sentence. State the null and alternative hypotheses, and identify which represents the claim.*

25. Light Bulbs A light bulb manufacturer claims that the mean life of a certain type of light bulb is more than 750 hours.

26. Shipping Errors As stated by a company's shipping department, the number of shipping errors per million shipments has a standard deviation that is less than 3.

27. Base Price of an ATV The standard deviation of the base price of a certain type of all-terrain vehicle is no more than $320.

28. Oak Trees A state park claims that the mean height of the oak trees in the park is at least 85 feet.

29. Drying Time A company claims that its brands of paint have a mean drying time of less than 45 minutes.

30. MP3 Players According to a recent survey, 74% of college students own an MP3 player. *(Source: Harris Interactive)*

Identifying Errors *In Exercises 31–36, write sentences describing type I and type II errors for a hypothesis test of the indicated claim.*

31. Repeat Buyers A furniture store claims that at least 60% of its new customers will return to buy their next piece of furniture.

32. Flow Rate A garden hose manufacturer advertises that the mean flow rate of a certain type of hose is 16 gallons per minute.

33. Chess A local chess club claims that the length of time to play a game has a standard deviation of more than 12 minutes.

34. Video Game Systems A researcher claims that the proportion of adults in the United States who own a video game system is not 26%.

35. Police A police station publicizes that at most 20% of applicants become police officers.

36. Computers A computer repairer advertises that the mean cost of removing a virus infection is less than $100.

Identifying Tests *In Exercises 37–42, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed. Explain your reasoning.*

37. Security Alarms At least 14% of all homeowners have a home security alarm.

38. Clocks A manufacturer of grandfather clocks claims that the mean time its clocks lose is no more than 0.02 second per day.

39. Golf The standard deviation of the 18-hole scores for a golfer is less than 2.1 strokes.

40. Lung Cancer A government report claims that the proportion of lung cancer cases that are due to smoking is 87%. *(Source: LungCancer.org)*

41. Baseball A baseball team claims that the mean length of its games is less than 2.5 hours.

42. Tuition A state claims that the mean tuition of its universities is no more than $25,000 per year.

Interpreting a Decision *In Exercises 43–48, consider each claim. If a hypothesis test is performed, how should you interpret a decision that*

(a) rejects the null hypothesis?

(b) fails to reject the null hypothesis?

43. Swans A scientist claims that the mean incubation period for swan eggs is less than 40 days.

44. Lawn Mowers The standard deviation of the life of a certain type of lawn mower is at most 2.8 years.

45. Hourly Wages The U.S. Department of Labor claims that the proportion of full-time workers earning over $450 per week is greater than 75%. *(Adapted from U.S. Bureau of Labor Statistics)*

46. Gas Mileage An automotive manufacturer claims the standard deviation for the gas mileage of its models is 3.9 miles per gallon.

47. Health Care Visits A researcher claims that the proportion of people who have had no health care visits in the past year is less than 17%. *(Adapted from National Center for Health Statistics)*

48. Calories A sports drink maker claims the mean calorie content of its beverages is 72 calories per serving.

49. Writing Hypotheses: Medicine Your medical research team is investigating the mean cost of a 30-day supply of a certain heart medication. A pharmaceutical company thinks that the mean cost is less than $60. You want to support this claim. How would you write the null and alternative hypotheses?

50. Writing Hypotheses: Taxicab Company A taxicab company claims that the mean travel time between two destinations is about 21 minutes. You work for the bus company and want to reject this claim. How would you write the null and alternative hypotheses?

51. Writing Hypotheses: Refrigerator Manufacturer A refrigerator manufacturer claims that the mean life of its competitor's refrigerators is less than 15 years. You are asked to perform a hypothesis test to test this claim. How would you write the null and alternative hypotheses if

(a) you represent the manufacturer and want to support the claim?

(b) you represent the competitor and want to reject the claim?

52. Writing Hypotheses: Internet Provider An Internet provider is trying to gain advertising deals and claims that the mean time a customer spends online per day is greater than 28 minutes. You are asked to test this claim. How would you write the null and alternative hypotheses if

(a) you represent the Internet provider and want to support the claim?

(b) you represent a competing advertiser and want to reject the claim?

EXTENDING CONCEPTS

53. Getting at the Concept Why can decreasing the probability of a type I error cause an increase in the probability of a type II error?

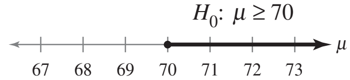
54. Getting at the Concept Explain why a level of significance of $\alpha = 0$ is not used.

55. Writing A null hypothesis is rejected with a level of significance of 0.05. Is it also rejected at a level of significance of 0.10? Explain.

56. Writing A null hypothesis is rejected with a level of significance of 0.10. Is it also rejected at a level of significance of 0.05? Explain.

Graphical Analysis *In Exercises 57–60, you are given a null hypothesis and three confidence intervals that represent three samplings. Decide whether each confidence interval indicates that you should reject H_0. Explain your reasoning.*

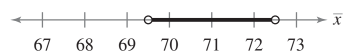
57. H_0: $\mu \geq 70$



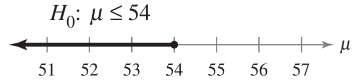
(a) $67 < \mu < 71$

(b) $67 < \mu < 69$

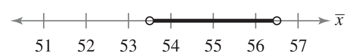
(c) $69.5 < \mu < 72.5$



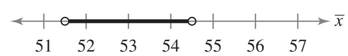
58. H_0: $\mu \leq 54$



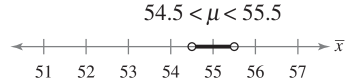
(a) $53.5 < \mu < 56.5$



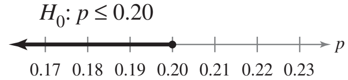
(b) $51.5 < \mu < 54.5$



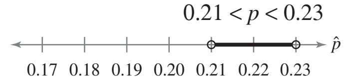
(c) $54.5 < \mu < 55.5$



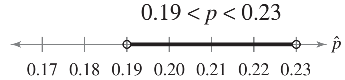
59. H_0: $p \leq 0.20$



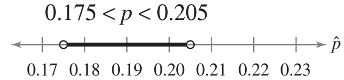
(a) $0.21 < p < 0.23$



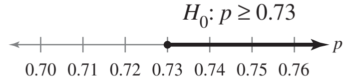
(b) $0.19 < p < 0.23$



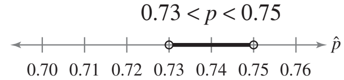
(c) $0.175 < p < 0.205$



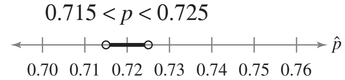
60. H_0: $p \geq 0.73$



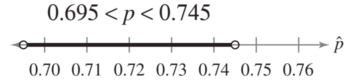
(a) $0.73 < p < 0.75$



(b) $0.715 < p < 0.725$



(c) $0.695 < p < 0.745$



7.2 Hypothesis Testing for the Mean (Large Samples)

WHAT YOU SHOULD LEARN

- How to find *P*-values and use them to test a mean μ
- How to use *P*-values for a *z*-test
- How to find critical values and rejection regions in a normal distribution
- How to use rejection regions for a *z*-test

Using *P*-Values to Make Decisions ▸ Using *P*-Values for a *z*-Test ▸ Rejection Regions and Critical Values ▸ Using Rejection Regions for a *z*-Test

▸ USING *P*-VALUES TO MAKE DECISIONS

In Chapter 5, you learned that when the sample size is at least 30, the sampling distribution for $\overline{x}$ (the sample mean) is normal. In Section 7.1, you learned that a way to reach a conclusion in a hypothesis test is to use a *P*-value for the sample statistic, such as $\overline{x}$. Recall that when you assume the null hypothesis is true, a *P*-value (or probability value) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data. The decision rule for a hypothesis test based on a *P*-value is as follows.

DECISION RULE BASED ON *P*-VALUE

To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

EXAMPLE 1

▸ Interpreting a *P*-Value

The *P*-value for a hypothesis test is $P = 0.0237$. What is your decision if the level of significance is (1) $\alpha = 0.05$ and (2) $\alpha = 0.01$?

▸ Solution

1. Because $0.0237 < 0.05$, you should reject the null hypothesis.

2. Because $0.0237 > 0.01$, you should fail to reject the null hypothesis.

▸ Try It Yourself 1

The *P*-value for a hypothesis test is $P = 0.0347$. What is your decision if the level of significance is (1) $\alpha = 0.01$ and (2) $\alpha = 0.05$?

a. *Compare* the *P*-value with the level of significance.
b. *Make* a decision.

Answer: Page A41

INSIGHT

The lower the *P*-value, the more evidence there is in favor of rejecting H_0. The *P*-value gives you the lowest level of significance for which the sample statistic allows you to reject the null hypothesis. In Example 1, you would reject H_0 at any level of significance greater than or equal to 0.0237.

FINDING THE *P*-VALUE FOR A HYPOTHESIS TEST

After determining the hypothesis test's standardized test statistic and the test statistic's corresponding area, do one of the following to find the *P*-value.

a. For a left-tailed test, $P =$ (Area in left tail).
b. For a right-tailed test, $P =$ (Area in right tail).
c. For a two-tailed test, $P = 2$(Area in tail of test statistic).

EXAMPLE 2

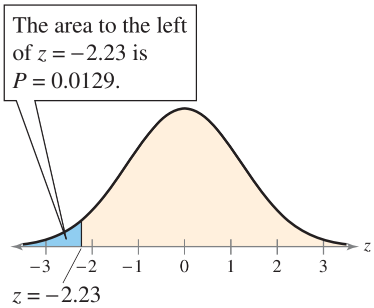
Finding a *P*-Value for a Left-Tailed Test

Find the P-value for a left-tailed hypothesis test with a test statistic of $z = -2.23$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.01$.

Solution

The graph shows a standard normal curve with a shaded area to the left of $z = -2.23$. For a left-tailed test,

$$P = (\text{Area in left tail}).$$



Left-Tailed Test

From Table 4 in Appendix B, the area corresponding to $z = -2.23$ is 0.0129, which is the area in the left tail. So, the P-value for a left-tailed hypothesis test with a test statistic of $z = -2.23$ is $P = 0.0129$.

Interpretation Because the P-value of 0.0129 is greater than 0.01, you should fail to reject H_0.

Try It Yourself 2

Find the P-value for a left-tailed hypothesis test with a test statistic of $z = -1.71$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.05$.

a. *Use* Table 4 in Appendix B to find the area that corresponds to $z = -1.71$.
b. *Calculate* the P-value for a left-tailed test, the area in the left tail.
c. *Compare* the P-value with α and *decide* whether to reject H_0.

Answer: Page A41

EXAMPLE 3

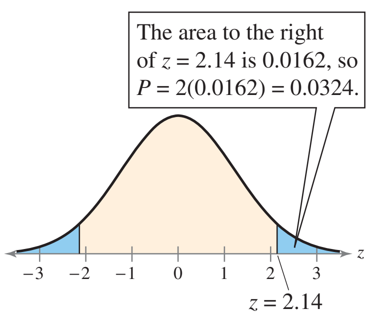
Finding a *P*-Value for a Two-Tailed Test

Find the P-value for a two-tailed hypothesis test with a test statistic of $z = 2.14$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.05$.

Solution

The graph shows a standard normal curve with shaded areas to the left of $z = -2.14$ and to the right of $z = 2.14$. For a two-tailed test,

$$P = 2(\text{Area in tail of test statistic}).$$



Two-Tailed Test

From Table 4, the area corresponding to $z = 2.14$ is 0.9838. The area in the right tail is $1 - 0.9838 = 0.0162$. So, the P-value for a two-tailed hypothesis test with a test statistic of $z = 2.14$ is

$$P = 2(0.0162) = 0.0324.$$

Interpretation Because the P-value of 0.0324 is less than 0.05, you should reject H_0.

Try It Yourself 3

Find the P-value for a two-tailed hypothesis test with a test statistic of $z = 1.64$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.10$.

a. *Use* Table 4 to find the area that corresponds to $z = 1.64$.
b. *Calculate* the P-value for a two-tailed test, twice the area in the tail of the test statistic.
c. *Compare* the P-value with α and *decide* whether to reject H_0.

Answer: Page A41

USING *P*-VALUES FOR A *z*-TEST

The z-test for the mean is used in populations for which the sampling distribution of sample means is normal. To use the z-test, you need to find the standardized value for your test statistic $\bar{x}$.

$$z = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}$$

STUDY TIP

With all hypothesis tests, it is helpful to sketch the sampling distribution. Your sketch should include the standardized test statistic.

z-TEST FOR A MEAN μ

The **z-test for a mean** is a statistical test for a population mean. The z-test can be used when the population is normal and σ is known, or for any population when the sample size n is at least 30. The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}.$$

Recall that $\dfrac{\sigma}{\sqrt{n}}$ = standard error $= \sigma_{\bar{x}}$.

When $n \geq 30$, you can use the sample standard deviation s in place of σ.

INSIGHT

When the sample size is at least 30, you know the following about the sampling distribution of sample means.

(1) The shape is normal.

(2) The mean is the hypothesized mean.

(3) The standard error is $s/\sqrt{n}$, where s is used in place of σ.

GUIDELINES

Using *P*-Values for a *z*-Test for Mean μ

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ or, if $n \geq 30$, use $\sigma \approx s$.
4. Find the area that corresponds to z.	Use Table 4 in Appendix B.

5. Find the P-value.

a. For a left-tailed test, P = (Area in left tail).

b. For a right-tailed test, P = (Area in right tail).

c. For a two-tailed test, P = 2(Area in tail of test statistic).

6. Make a decision to reject or fail to reject the null hypothesis.	Reject H_0 if P-value is less than or equal to α. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.	

EXAMPLE 4

Hypothesis Testing Using *P*-Values

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random selection of 32 pit stop times has a sample mean of 12.9 seconds and a standard deviation of 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a *P*-value.

Solution

The claim is "the mean pit stop time is less than 13 seconds." So, the null and alternative hypotheses are

H_0: $\mu \geq 13$ seconds and H_a: $\mu < 13$ seconds. (Claim)

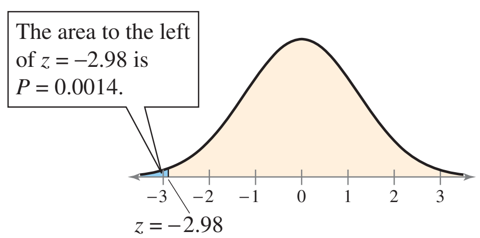
The level of significance is $\alpha = 0.01$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because $n \geq 30$, use the z-test.

$$\approx \frac{12.9 - 13}{0.19/\sqrt{32}}$$ Because $n \geq 30$, use $\sigma \approx s = 0.19$. Assume $\mu = 13$.

$$\approx -2.98.$$

In Table 4 in Appendix B, the area corresponding to $z = -2.98$ is 0.0014. Because this test is a left-tailed test, the *P*-value is equal to the area to the left of $z = -2.98$. So, $P = 0.0014$. Because the *P*-value is less than $\alpha = 0.01$, you should decide to reject the null hypothesis.



Left-Tailed Test

Interpretation There is enough evidence at the 1% level of significance to support the claim that the mean pit stop time is less than 13 seconds.

Try It Yourself 4

Homeowners claim that the mean speed of automobiles traveling on their street is greater than the speed limit of 35 miles per hour. A random sample of 100 automobiles has a mean speed of 36 miles per hour and a standard deviation of 4 miles per hour. Is there enough evidence to support the claim at $\alpha = 0.05$? Use a *P*-value.

a. Identify the *claim*. Then state the *null* and *alternative hypotheses*.
b. Identify the *level of significance*.
c. Find the *standardized test statistic* z.
d. Find the *P-value*.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A41

EXAMPLE 5

SC Report 29

See MINITAB steps on page 424.

Hypothesis Testing Using *P*-Values

The National Institute of Diabetes and Digestive and Kidney Diseases reports that the average cost of bariatric (weight loss) surgery is about \$22,500. You think this information is incorrect. You randomly select 30 bariatric surgery patients and find that the average cost for their surgeries is \$21,545 with a standard deviation of \$3015. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P-value. *(Adapted from National Institute of Diabetes and Digestive and Kidney Diseases)*

Solution

The claim is "the mean is different from \$22,500." So, the null and alternative hypotheses are

$$H_0: \mu = \$22{,}500$$

and

$$H_a: \mu \neq \$22{,}500. \text{ (Claim)}$$

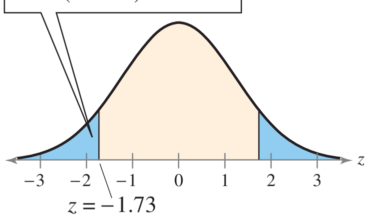
The level of significance is $\alpha = 0.05$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{Because } n \geq 30, \text{ use the } z\text{-test.}$$

$$\approx \frac{21{,}545 - 22{,}500}{3015/\sqrt{30}} \qquad \text{Because } n \geq 30, \text{ use } \sigma \approx s = 3015. \text{ Assume } \mu = 22{,}500.$$

$$\approx -1.73.$$

The area to the left of $z = -1.73$ is 0.0418, so $P = 2(0.0418) = 0.0836$.



Two-Tailed Test

In Table 4, the area corresponding to $z = -1.73$ is 0.0418. Because the test is a two-tailed test, the P-value is equal to twice the area to the left of $z = -1.73$. So,

$$P = 2(0.0418)$$
$$= 0.0836.$$

Because the P-value is greater than α, you should fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 5% level of significance to support the claim that the mean cost of bariatric surgery is different from \$22,500.

Try It Yourself 5

One of your distributors reports an average of 150 sales per day. You suspect that this average is not accurate, so you randomly select 35 days and determine the number of sales each day. The sample mean is 143 daily sales with a standard deviation of 15 sales. At $\alpha = 0.01$, is there enough evidence to doubt the distributor's reported average? Use a P-value.

a. Identify the *claim*. Then state the *null* and *alternative hypotheses*.
b. Identify the *level of significance*.
c. Find the *standardized test statistic* z.
d. Find the *P-value*.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

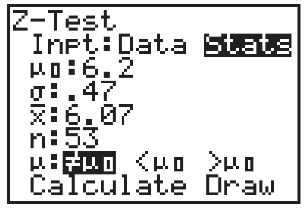
Answer: Page A41

EXAMPLE 6

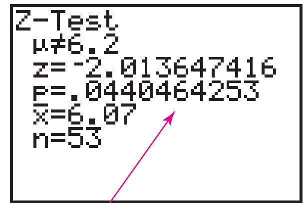
Using a Technology Tool to Find a *P*-Value

What decision should you make for the following TI-83/84 Plus displays, using a level of significance of $\alpha = 0.05$?

TI-83/84 Plus



TI-83/84 Plus



Solution

The *P*-value for this test is given as 0.0440464253. Because the *P*-value is less than 0.05, you should reject the null hypothesis.

Try It Yourself 6

For the TI-83/84 Plus hypothesis test shown in Example 6, make a decision at the $\alpha = 0.01$ level of significance.

a. *Compare* the *P*-value with the level of significance.
b. *Make* your decision.

Answer: Page A41

STUDY TIP

Using a TI-83/84 Plus, you can either enter the original data into a list to find a *P*-value or enter the descriptive statistics.

STAT

Choose the TESTS menu.

1: Z-Test…

Select the *Data* input option if you use the original data. Select the *Stats* input option if you use the descriptive statistics. In each case, enter the appropriate values including the corresponding type of hypothesis test indicated by the alternative hypothesis. Then select *Calculate*.

REJECTION REGIONS AND CRITICAL VALUES

Another method to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the rejection region of the sampling distribution.

INSIGHT

If the test statistic falls in a rejection region, it would be considered an unusual event.

DEFINITION

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a test statistic falls in this region, the null hypothesis is rejected. A **critical value** z_0 separates the rejection region from the nonrejection region.

GUIDELINES

Finding Critical Values in a Normal Distribution

1. Specify the level of significance α.
2. Decide whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0. If the hypothesis test is
   a. *left-tailed*, find the z-score that corresponds to an area of α.
   b. *right-tailed*, find the z-score that corresponds to an area of $1 - \alpha$.
   c. *two-tailed*, find the z-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s).

If you cannot find the exact area in Table 4, use the area that is closest. When the area is exactly midway between two areas in the table, use the z-score midway between the corresponding z-scores.

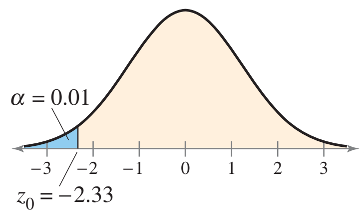
EXAMPLE 7

Finding a Critical Value for a Left-Tailed Test

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.01$.

Solution

The graph shows a standard normal curve with a shaded area of 0.01 in the left tail. In Table 4, the z-score that is closest to an area of 0.01 is -2.33. So, the critical value is $z_0 = -2.33$. The rejection region is to the left of this critical value.



1% Level of Significance

Try It Yourself 7

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.10$.

a. *Draw* a graph of the standard normal curve with an area of α in the left tail.
b. *Use* Table 4 to find the area that is closest to α.
c. *Find* the z-score that corresponds to this area.
d. *Identify* the rejection region.

Answer: Page A41

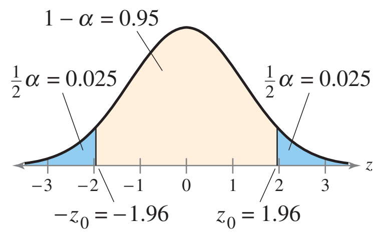
EXAMPLE 8

Finding a Critical Value for a Two-Tailed Test

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.05$.

Solution

The graph shows a standard normal curve with shaded areas of $\frac{1}{2}\alpha = 0.025$ in each tail. The area to the left of $-z_0$ is $\frac{1}{2}\alpha = 0.025$, and the area to the left of z_0 is $1 - \frac{1}{2}\alpha = 0.975$. In Table 4, the z-scores that correspond to the areas 0.025 and 0.975 are -1.96 and 1.96, respectively. So, the critical values are $-z_0 = -1.96$ and $z_0 = 1.96$. The rejection regions are to the left of -1.96 and to the right of 1.96.



5% Level of Significance

STUDY TIP

Notice in Example 8 that the critical values are opposites. This is always true for two-tailed z-tests.

The table lists the critical values for commonly used levels of significance.

Alpha	Tail	z
0.10	Left	-1.28
	Right	1.28
	Two	± 1.645
0.05	Left	-1.645
	Right	1.645
	Two	± 1.96
0.01	Left	-2.33
	Right	2.33
	Two	± 2.575

Try It Yourself 8

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.08$.

a. *Draw* a graph of the standard normal curve with an area of $\frac{1}{2}\alpha$ in each tail.
b. *Use* Table 4 to find the areas that are closest to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
c. *Find* the z-scores that correspond to these areas.
d. *Identify* the rejection regions.

Answer: Page A41

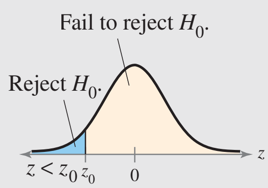
USING REJECTION REGIONS FOR A z-TEST

To conclude a hypothesis test using rejection region(s), you make a decision and interpret the decision as follows.

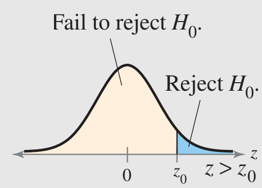
DECISION RULE BASED ON REJECTION REGION

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic z. If the standardized test statistic

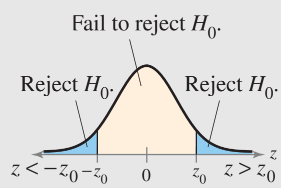
1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

Failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis.

GUIDELINES

Using Rejection Regions for a z-Test for a Mean μ

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the critical value(s).	Use Table 4 in Appendix B.
4. Determine the rejection region(s).	
5. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$, or, if $n \geq 30$, use $\sigma \approx s$.
6. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.	

PICTURING THE WORLD

Each year, the Environmental Protection Agency (EPA) publishes reports of gas mileage for all makes and models of passenger vehicles. In a recent year, small station wagons with automatic transmissions that posted the best mileage were the Audi A3 (diesel) and the Volkswagen Jetta SportWagen (diesel). Each had a mean mileage of 30 miles per gallon (city) and 42 miles per gallon (highway). Suppose that Volkswagen believes a Jetta SportWagen exceeds 42 miles per gallon on the highway. To support its claim, it tests 36 vehicles on highway driving and obtains a sample mean of 43.2 miles per gallon with a standard deviation of 2.1 miles per gallon. (Source: U.S. Department of Energy)

Is the evidence strong enough to support the claim that the Jetta SportWagen's highway miles per gallon exceeds the EPA estimate? Use a z-test with $\alpha = 0.01$.

EXAMPLE 9

See TI-83/84 Plus steps on page 425.

Testing μ with a Large Sample

Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of one of its competitors, which is \$68,000. A random sample of 30 of the company's mechanical engineers has a mean salary of \$66,900 with a standard deviation of \$5500. At $\alpha = 0.05$, test the employees' claim.

Solution

The claim is "the mean salary is less than \$68,000." So, the null and alternative hypotheses can be written as

$$H_0: \mu \geq \$68{,}000 \quad \text{and} \quad H_a: \mu < \$68{,}000. \quad \text{(Claim)}$$

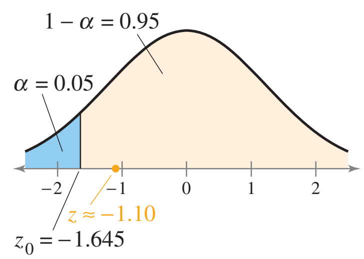
Because the test is a left-tailed test and the level of significance is $\alpha = 0.05$, the critical value is $z_0 = -1.645$ and the rejection region is $z < -1.645$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{Because } n \geq 30 \text{, use the } z\text{-test.}$$

$$\approx \frac{66{,}900 - 68{,}000}{5500/\sqrt{30}} \qquad \text{Because } n \geq 30 \text{, use } \sigma \approx s = 5500. \text{ Assume } \mu = 68{,}000.$$

$$\approx -1.10.$$

The graph shows the location of the rejection region and the standardized test statistic z. Because z is not in the rejection region, you fail to reject the null hypothesis.



5% Level of Significance

Interpretation There is not enough evidence at the 5% level of significance to support the employees' claim that the mean salary is less than \$68,000.

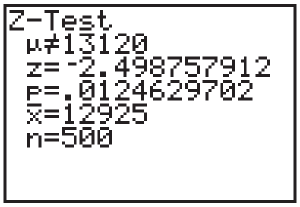
Be sure you understand the decision made in this example. Even though your sample has a mean of \$66,900, you cannot (at a 5% level of significance) support the claim that the mean of all the mechanical engineers' salaries is less than \$68,000. The difference between your test statistic and the hypothesized mean is probably due to sampling error.

Try It Yourself 9

The CEO of the company claims that the mean work day of the company's mechanical engineers is less than 8.5 hours. A random sample of 35 of the company's mechanical engineers has a mean work day of 8.2 hours with a standard deviation of 0.5 hour. At $\alpha = 0.01$, test the CEO's claim.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α.
c. Find the *critical value* z_0 and identify the *rejection region*.
d. Find the *standardized test statistic* z. *Sketch* a graph.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A41



Using a TI-83/84 Plus, you can find the standardized test statistic automatically.

EXAMPLE 10

Testing μ with a Large Sample

The U.S. Department of Agriculture claims that the mean cost of raising a child from birth to age 2 by husband-wife families in the United States is \$13,120. A random sample of 500 children (age 2) has a mean cost of \$12,925 with a standard deviation of \$1745. At $\alpha = 0.10$, is there enough evidence to reject the claim? *(Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion)*

Solution

The claim is "the mean cost is \$13,120." So, the null and alternative hypotheses are

H_0: $\mu = \$13{,}120$ (Claim)

and

H_a: $\mu \neq \$13{,}120$.

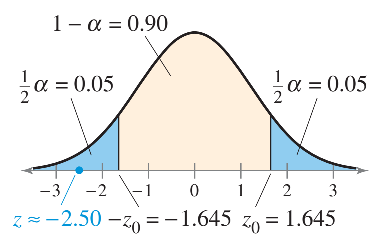
Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because $n \geq 30$, use the z-test.

$$\approx \frac{12{,}925 - 13{,}120}{1745/\sqrt{500}}$$ Because $n \geq 30$, use $\sigma \approx s = 1745$. Assume $\mu = 13{,}120$.

$$\approx -2.50.$$

The graph shows the location of the rejection regions and the standardized test statistic z. Because z is in the rejection region, you should reject the null hypothesis.



5% Level of Significance

Interpretation There is enough evidence at the 10% level of significance to reject the claim that the mean cost of raising a child from birth to age 2 by husband-wife families in the United States is \$13,120.

Try It Yourself 10

Using the information and results of Example 10, determine whether there is enough evidence to reject the claim that the mean cost of raising a child from birth to age 2 by husband-wife families in the United States is \$13,120. Use $\alpha = 0.01$.

a. Identify the *level of significance* α.
b. Find the *critical values* $-z_0$ and z_0 and identify the *rejection regions*.
c. *Sketch* a graph. *Decide* whether to reject the null hypothesis.
d. *Interpret* the decision in the context of the original claim.

Answer: Page A41

7.2 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. Explain the difference between the z-test for μ using rejection region(s) and the z-test for μ using a P-value.

2. In hypothesis testing, does choosing between the critical value method or the P-value method affect your conclusion? Explain.

In Exercises 3–8, find the P-value for the indicated hypothesis test with the given standardized test statistic z. Decide whether to reject H_0 for the given level of significance α.

3. Left-tailed test, $z = -1.32$, $\alpha = 0.10$

4. Left-tailed test, $z = -1.55$, $\alpha = 0.05$

5. Right-tailed test, $z = 2.46$, $\alpha = 0.01$

6. Right-tailed test, $z = 1.23$, $\alpha = 0.10$

7. Two-tailed test, $z = -1.68$, $\alpha = 0.05$

8. Two-tailed test, $z = 2.30$, $\alpha = 0.01$

Graphical Analysis *In Exercises 9–12, match each P-value with the graph that displays its area. The graphs are labeled (a)–(d).*

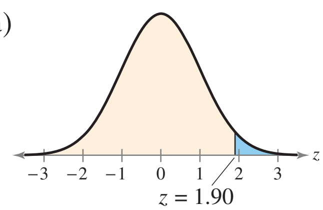
9. $P = 0.0089$

10. $P = 0.3050$

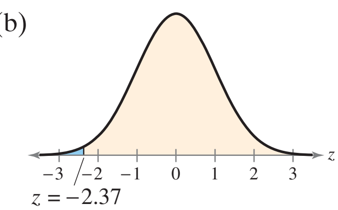
11. $P = 0.0688$

12. $P = 0.0287$

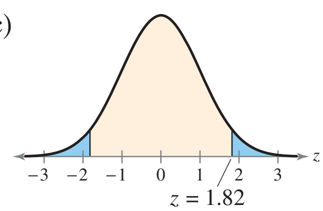
(a)



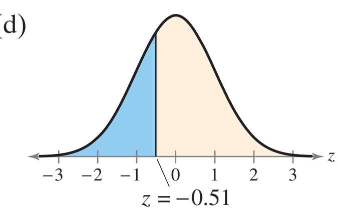
(b)



(c)



(d)



13. Given H_0: $\mu = 100$, H_a: $\mu \neq 100$, and $P = 0.0461$.

(a) Do you reject or fail to reject H_0 at the 0.01 level of significance?

(b) Do you reject or fail to reject H_0 at the 0.05 level of significance?

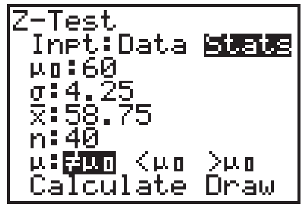
14. Given H_0: $\mu \geq 8.5$, H_a: $\mu < 8.5$, and $P = 0.0691$.

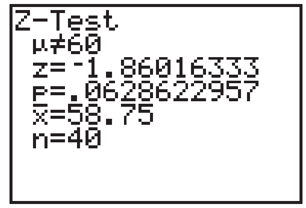
(a) Do you reject or fail to reject H_0 at the 0.01 level of significance?

(b) Do you reject or fail to reject H_0 at the 0.05 level of significance?

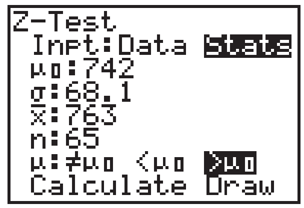
In Exercises 15 and 16, use the TI-83/84 Plus displays to make a decision to reject or fail to reject the null hypothesis at the given level of significance.

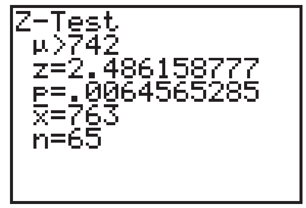
15. $\alpha = 0.05$





16. $\alpha = 0.01$





Finding Critical Values *In Exercises 17–22, find the critical value(s) for the indicated type of test and level of significance α. Include a graph with your answer.*

17. Right-tailed test, $\alpha = 0.05$

18. Right-tailed test, $\alpha = 0.08$

19. Left-tailed test, $\alpha = 0.03$

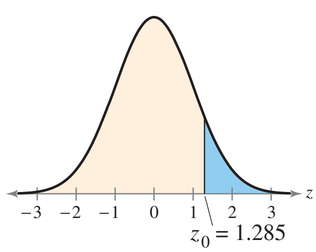
20. Left-tailed test, $\alpha = 0.09$

21. Two-tailed test, $\alpha = 0.02$

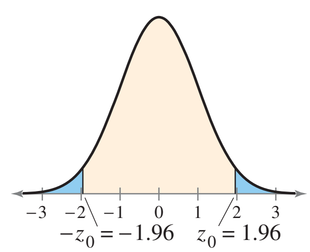
22. Two-tailed test, $\alpha = 0.10$

Graphical Analysis *In Exercises 23 and 24, state whether each standardized test statistic z allows you to reject the null hypothesis. Explain your reasoning.*

23. (a) $z = -1.301$
(b) $z = 1.203$
(c) $z = 1.280$
(d) $z = 1.286$



24. (a) $z = 1.98$
(b) $z = -1.89$
(c) $z = 1.65$
(d) $z = -1.99$



In Exercises 25–28, test the claim about the population mean μ at the given level of significance α using the given sample statistics.

25. Claim: $\mu = 40$; $\alpha = 0.05$.
Sample statistics: $\bar{x} = 39.2$, $s = 3.23$, $n = 75$

26. Claim: $\mu > 1745$; $\alpha = 0.10$.
Sample statistics: $\bar{x} = 1752$, $s = 38$, $n = 44$

27. Claim: $\mu \neq 8550$; $\alpha = 0.02$.
Sample statistics: $\bar{x} = 8420$, $s = 314$, $n = 38$

28. Claim: $\mu \leq 22{,}500$; $\alpha = 0.01$.
Sample statistics: $\bar{x} = 23{,}250$, $s = 1200$, $n = 45$

USING AND INTERPRETING CONCEPTS

Testing Claims Using *P*-Values *In Exercises 29–34,*

(a) write the claim mathematically and identify H_0 and H_a.

(b) find the standardized test statistic z and its corresponding area. If convenient, use technology.

(c) find the P-value. If convenient, use technology.

(d) decide whether to reject or fail to reject the null hypothesis.

(e) interpret the decision in the context of the original claim.

29. MCAT Scores A random sample of 50 medical school applicants at a university has a mean raw score of 31 with a standard deviation of 2.5 on the multiple choice portions of the Medical College Admission Test (MCAT). A student says that the mean raw score for the school's applicants is more than 30. At $\alpha = 0.01$, is there enough evidence to support the student's claim? *(Adapted from Association of American Medical Colleges)*

30. Sprinkler Systems A manufacturer of sprinkler systems designed for fire protection claims that the average activating temperature is at least 135°F. To test this claim, you randomly select a sample of 32 systems and find the mean activation temperature to be 133°F with a standard deviation of 3.3°F. At $\alpha = 0.10$, do you have enough evidence to reject the manufacturer's claim?

31. Bottled Water Consumption The U.S. Department of Agriculture claims that the mean consumption of bottled water by a person in the United States is 28.5 gallons per year. A random sample of 100 people in the United States has a mean bottled water consumption of 27.8 gallons per year with a standard deviation of 4.1 gallons. At $\alpha = 0.08$, can you reject the claim? *(Adapted from U.S. Department of Agriculture)*

32. Coffee Consumption The U.S. Department of Agriculture claims that the mean consumption of coffee by a person in the United States is 24.2 gallons per year. A random sample of 120 people in the United States shows that the mean coffee consumption is 23.5 gallons per year with a standard deviation of 3.2 gallons. At $\alpha = 0.05$, can you reject the claim? *(Adapted from U.S. Department of Agriculture)*

33. Quitting Smoking The lengths of time (in years) it took a random sample of 32 former smokers to quit smoking permanently are listed. At $\alpha = 0.05$, is there enough evidence to reject the claim that the mean time it takes smokers to quit smoking permanently is 15 years? *(Adapted from The Gallup Organization)*

15.7	13.2	22.6	13.0	10.7	18.1	14.7	7.0	17.3	7.5	21.8
12.3	19.8	13.8	16.0	15.5	13.1	20.7	15.5	9.8	11.9	16.9
7.0	19.3	13.2	14.6	20.9	15.4	13.3	11.6	10.9	21.6	

34. Salaries An analyst claims that the mean annual salary for advertising account executives in Denver, Colorado is more than the national mean, \$66,200. The annual salaries (in dollars) for a random sample of 35 advertising account executives in Denver are listed. At $\alpha = 0.09$, is there enough evidence to support the analyst's claim? *(Adapted from Salary.com)*

69,450	65,910	68,780	66,724	64,125	67,561	62,419
70,375	65,835	62,653	65,090	67,997	65,176	64,936
66,716	69,832	63,111	64,550	63,512	65,800	66,150
68,587	68,276	65,902	63,415	64,519	70,275	70,102
67,230	65,488	66,225	69,879	69,200	65,179	69,755

Testing Claims Using Critical Values *In Exercises 35–42, (a) write the claim mathematically and identify H_0 and H_a, (b) find the critical values and identify the rejection regions, (c) find the standardized test statistic, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.*

35. Caffeine Content in Colas A company that makes cola drinks states that the mean caffeine content per 12-ounce bottle of cola is 40 milligrams. You want to test this claim. During your tests, you find that a random sample of thirty 12-ounce bottles of cola has a mean caffeine content of 39.2 milligrams with a standard deviation of 7.5 milligrams. At $\alpha = 0.01$, can you reject the company's claim? *(Adapted from American Beverage Association)*

36. Electricity Consumption The U.S. Energy Information Association claims that the mean monthly residential electricity consumption in your town is 874 kilowatt-hours (kWh). You want to test this claim. You find that a random sample of 64 residential customers has a mean monthly electricity consumption of 905 kWh and a standard deviation of 125 kWh. At $\alpha = 0.05$, do you have enough evidence to reject the association's claim? *(Adapted from U.S. Energy Information Association)*

37. Light Bulbs A light bulb manufacturer guarantees that the mean life of a certain type of light bulb is at least 750 hours. A random sample of 36 light bulbs has a mean life of 745 hours with a standard deviation of 60 hours. At $\alpha = 0.02$, do you have enough evidence to reject the manufacturer's claim?

38. Fast Food A fast food restaurant estimates that the mean sodium content in one of its breakfast sandwiches is no more than 920 milligrams. A random sample of 44 breakfast sandwiches has a mean sodium content of 925 with a standard deviation of 18 milligrams. At $\alpha = 0.10$, do you have enough evidence to reject the restaurant's claim?

39. Nitrogen Dioxide Levels A scientist estimates that the mean nitrogen dioxide level in Calgary is greater than 32 parts per billion. You want to test this estimate. To do so, you determine the nitrogen dioxide levels for 34 randomly selected days. The results (in parts per billion) are listed below. At $\alpha = 0.06$, can you support the scientist's estimate? *(Adapted from Clean Air Strategic Alliance)*

24	36	44	35	44	34	29	40	39	43	41	32
33	29	29	43	25	39	25	42	29	22	22	25
14	15	14	29	25	27	22	24	18	17		

40. Fluorescent Lamps A fluorescent lamp manufacturer guarantees that the mean life of a certain type of lamp is at least 10,000 hours. You want to test this guarantee. To do so, you record the lives of a random sample of 32 fluorescent lamps. The results (in hours) are shown below. At $\alpha = 0.09$, do you have enough evidence to reject the manufacturer's claim?

8,800	9,155	13,001	10,250	10,002	11,413	8,234	10,402
10,016	8,015	6,110	11,005	11,555	9,254	6,991	12,006
10,420	8,302	8,151	10,980	10,186	10,003	8,814	11,445
6,277	8,632	7,265	10,584	9,397	11,987	7,556	10,380

41. Weight Loss A weight loss program claims that program participants have a mean weight loss of at least 10 pounds after 1 month. You work for a medical association and are asked to test this claim. A random sample of 30 program participants and their weight losses (in pounds) after 1 month is listed in the stem-and-leaf plot at the left. At $\alpha = 0.03$, do you have enough evidence to reject the program's claim?

Weight Loss (in pounds) after One Month

5	7 7	Key: 5 | 7 = 5.7
6	6 7	
7	0 1 9	
8	2 2 7 9	
9	0 3 5 6 8	
10	2 5 6 6	
11	1 2 5 7 8	
12	0 7 8	
13	8	
14		
15	0	

FIGURE FOR EXERCISE 41

Evacuation Time (in seconds)

0	7 9
1	1 9 9
2	2 6 7 9 9
3	1 1 6 7 7 9 9
4	1 1 3 3 3 4 6 6 7
5	2 3 4 5 7 8 8 8 9 9
6	1 3 3 4 6 6 7
7	4 6 9
8	4 6
9	4
10	2

Key: 0|7 = 7

FIGURE FOR EXERCISE 42

42. Fire Drill An engineering company claims that the mean time it takes an employee to evacuate a building during a fire drill is less than 60 seconds. You want to test this claim. A random sample of 50 employees and their evacuation times (in seconds) is listed in the stem-and-leaf plot at the left. At $\alpha = 0.01$, can you support the company's claim?

SC *In Exercises 43–46, use StatCrunch to help you test the claim about the population mean μ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed.*

43. Claim: $\mu = 58$; $\alpha = 0.10$. Sample statistics: $\bar{x} = 57.6$, $s = 2.35$, $n = 80$

44. Claim: $\mu > 495$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 498.4$, $s = 17.8$, $n = 65$

45. Claim: $\mu \le 1210$; $\alpha = 0.08$. Sample statistics: $\bar{x} = 1234.21$, $s = 205.87$, $n = 250$

46. Claim: $\mu \ne 28{,}750$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 29{,}130$, $s = 3200$, $n = 600$

EXTENDING CONCEPTS

47. Water Usage You believe the mean annual water usage of U.S. households is less than 127,400 gallons. You find that a random sample of 30 households has a mean water usage of 125,270 gallons with a standard deviation of 6275 gallons. You conduct a statistical experiment where H_0: $\mu \ge 127{,}400$ and H_a: $\mu < 127{,}400$. At $\alpha = 0.01$, explain why you cannot reject H_0. *(Adapted from American Water Works Association)*

48. Vehicle Miles of Travel You believe the annual mean vehicle miles of travel (VMT) per U.S. household is greater than 22,000 miles. You do some research and find that a random sample of 36 U.S. households has a mean annual VMT of 22,200 miles with a standard deviation of 775 miles. You conduct a statistical experiment where H_0: $\mu \le 22{,}000$ and H_a: $\mu > 22{,}000$. At $\alpha = 0.05$, explain why you cannot reject H_0. *(Adapted from U.S. Federal Highway Administration)*

49. Using Different Values of α and n In Exercise 47, you believe that H_0 is not valid. Which of the following allows you to reject H_0? Explain your reasoning.

(a) Use the same values but increase α from 0.01 to 0.02.

(b) Use the same values but increase α from 0.01 to 0.05.

(c) Use the same values but increase n from 30 to 40.

(d) Use the same values but increase n from 30 to 50.

50. Using Different Values of α and n In Exercise 48, you believe that H_0 is not valid. Which of the following allows you to reject H_0? Explain your reasoning.

(a) Use the same values but increase α from 0.05 to 0.06.

(b) Use the same values but increase α from 0.05 to 0.07.

(c) Use the same values but increase n from 36 to 40.

(d) Use the same values but increase n from 36 to 80.

CASE STUDY

Human Body Temperature: What's Normal?

In an article in the *Journal of Statistics Education* (vol. 4, no. 2), Allen Shoemaker describes a study that was reported in the Journal of the American Medical Association (JAMA).* It is generally accepted that the mean body temperature of an adult human is 98.6°F. In his article, Shoemaker uses the data from the JAMA article to test this hypothesis. Here is a summary of his test.

Claim: The body temperature of adults is 98.6°F.

H_0: $\mu = 98.6$°F (Claim) $\qquad H_a$: $\mu \neq 98.6$°F

Sample Size: $n = 130$

Population: Adult human temperatures (Fahrenheit)

Distribution: Approximately normal

Test Statistics: $\bar{x} = 98.25$, $s = 0.73$

* Data for the JAMA article were collected from healthy men and women, ages 18 to 40, at the University of Maryland Center for Vaccine Development, Baltimore.

Men's Temperatures (in degrees Fahrenheit)

96	3
96	7 9
97	0 1 1 1 2 3 4 4 4 4
97	5 5 6 6 6 7 8 8 8 8 9 9
98	0 0 0 0 0 0 1 1 2 2 2 2 3 3 4 4 4 4
98	5 5 6 6 6 6 6 6 7 7 8 8 8 9
99	0 0 0 1 2 3 4
99	5
100	
100	

Key: 96|3 = 96.3

Women's Temperatures (in degrees Fahrenheit)

96	4
96	7 8
97	2 2 4
97	6 7 7 8 8 8 9 9 9
98	0 0 0 0 0 1 2 2 2 2 2 2 3 3 3 4 4 4 4 4
98	5 6 6 6 6 7 7 7 7 7 7 8 8 8 8 8 8 8 9
99	0 0 1 1 2 2 3 4
99	9
100	0
100	8

Key: 96|4 = 96.4

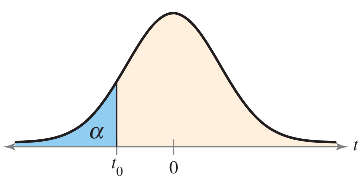
EXERCISES

1. Complete the hypothesis test for all adults (men and women) by performing the following steps. Use a level of significance of $\alpha = 0.05$.
   (a) Sketch the sampling distribution.
   (b) Determine the critical values and add them to your sketch.
   (c) Determine the rejection regions and shade them in your sketch.
   (d) Find the standardized test statistic. Add it to your sketch.
   (e) Make a decision to reject or fail to reject the null hypothesis.
   (f) Interpret the decision in the context of the original claim.
2. If you lower the level of significance to $\alpha = 0.01$, does your decision change? Explain your reasoning.
3. Test the hypothesis that the mean temperature of men is 98.6°F. What can you conclude at a level of significance of $\alpha = 0.01$?
4. Test the hypothesis that the mean temperature of women is 98.6°F. What can you conclude at a level of significance of $\alpha = 0.01$?
5. Use the sample of 130 temperatures to form a 99% confidence interval for the mean body temperature of adult humans.
6. The conventional "normal" body temperature was established by Carl Wunderlich over 100 years ago. What were possible sources of error in Wunderlich's sampling procedure?

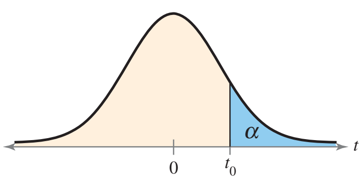
7.3 Hypothesis Testing for the Mean (Small Samples)

WHAT YOU SHOULD LEARN

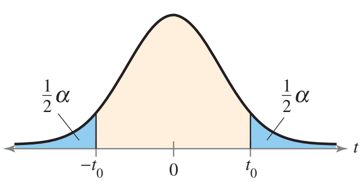
- How to find critical values in a t-distribution
- How to use the t-test to test a mean μ
- How to use technology to find P-values and use them with a t-test to test a mean μ



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

Critical Values in a t-Distribution ▸ The t-Test for a Mean μ ($n < 30$, σ unknown) ▸ Using P-Values with t-Tests

▸ CRITICAL VALUES IN A *t*-DISTRIBUTION

In Section 7.2, you learned how to perform a hypothesis test for a population mean when the sample size was at least 30. In real life, it is often not practical to collect samples of size 30 or more. However, if the population has a normal, or nearly normal, distribution, you can still test the population mean μ. To do so, you can use the t-sampling distribution with $n - 1$ degrees of freedom.

GUIDELINES

Finding Critical Values in a *t*-Distribution

1. Identify the level of significance α.
2. Identify the degrees of freedom d.f. $= n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. If the hypothesis test is
   - **a.** *left-tailed*, use the "One Tail, α" column with a negative sign.
   - **b.** *right-tailed*, use the "One Tail, α" column with a positive sign.
   - **c.** *two-tailed*, use the "Two Tails, α" column with a negative and a positive sign.

EXAMPLE 1

▸ Finding Critical Values for *t*

Find the critical value t_0 for a left-tailed test with $\alpha = 0.05$ and $n = 21$.

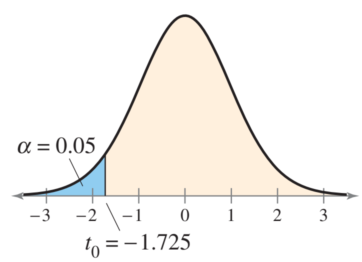
▸ Solution

The degrees of freedom are

$$\begin{aligned} \text{d.f.} &= n - 1 \\ &= 21 - 1 \\ &= 20. \end{aligned}$$

To find the critical value, use Table 5 in Appendix B with d.f. $= 20$ and $\alpha = 0.05$ in the "One Tail, α" column. Because the test is a left-tailed test, the critical value is negative. So,

$$t_0 = -1.725.$$



5% Level of Significance

▸ Try It Yourself 1

Find the critical value t_0 for a left-tailed test with $\alpha = 0.01$ and $n = 14$.

a. Identify the *degrees of freedom*.
b. *Use* the "One Tail, α" column in Table 5 in Appendix B to find t_0.

Answer: Page A41

EXAMPLE 2

Finding Critical Values for t

Find the critical value t_0 for a right-tailed test with $\alpha = 0.01$ and $n = 17$.

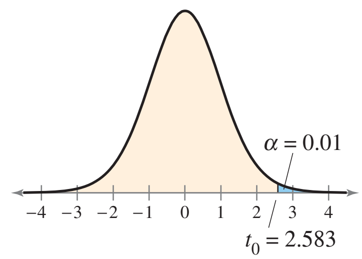
Solution

The degrees of freedom are

$$\begin{aligned} \text{d.f.} &= n - 1 \\ &= 17 - 1 \\ &= 16. \end{aligned}$$

To find the critical value, use Table 5 with d.f. = 16 and $\alpha = 0.01$ in the "One Tail, α" column. Because the test is right-tailed, the critical value is positive. So,

$$t_0 = 2.583.$$



1% Level of Significance

Try It Yourself 2

Find the critical value t_0 for a right-tailed test with $\alpha = 0.10$ and $n = 9$.

a. Identify the *degrees of freedom*.
b. *Use* the "One Tail, α" column in Table 5 in Appendix B to find t_0.

Answer: Page A41

EXAMPLE 3

Finding Critical Values for t

Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.10$ and $n = 26$.

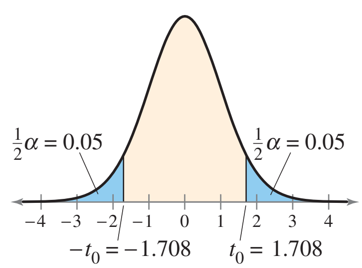
Solution

The degrees of freedom are

$$\begin{aligned} \text{d.f.} &= n - 1 \\ &= 26 - 1 \\ &= 25. \end{aligned}$$

To find the critical values, use Table 5 with d.f. = 25 and $\alpha = 0.10$ in the "Two Tails, α" column. Because the test is two-tailed, one critical value is negative and one is positive. So,

$$-t_0 = -1.708 \quad \text{and} \quad t_0 = 1.708.$$



10% Level of Significance

Try It Yourself 3

Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.05$ and $n = 16$.

a. Identify the *degrees of freedom*.
b. *Use* the "Two Tails, α" column in Table 5 in Appendix B to find t_0.

Answer: Page A41

PICTURING THE WORLD

On the basis of a t-test, a decision was made whether to send truckloads of waste contaminated with cadmium to a sanitary landfill or a hazardous waste landfill. The trucks were sampled to determine if the mean level of cadmium exceeded the allowable amount of 1 milligram per liter for a sanitary landfill. Assume the null hypothesis was $\mu \leq 1$. (Adapted from Pacific Northwest National Laboratory)

	H_0 True	H_0 False
Fail to reject H_0.		
Reject H_0.		

Describe the possible type I and type II errors of this situation.

THE t-TEST FOR A MEAN μ ($n < 30$, σ UNKNOWN)

To test a claim about a mean μ using a small sample $(n < 30)$ from a normal, or nearly normal, distribution when σ is unknown, you can use a t-sampling distribution.

$$t = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}$$

t-TEST FOR A MEAN μ

The ***t*-test for a mean** is a statistical test for a population mean. The t-test can be used when the population is normal or nearly normal, σ is unknown, and $n < 30$. The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}.$$

The degrees of freedom are

$$\text{d.f.} = n - 1.$$

GUIDELINES

Using the t-Test for a Mean μ (Small Sample)

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom.	$\text{d.f.} = n - 1$
4. Determine the critical value(s).	Use Table 5 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If t is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Remember that when you make a decision, the possibility of a type I or a type II error exists.

If you prefer using P-values, turn to page 392 to learn how to use P-values for a t-test for a mean μ (small sample).

EXAMPLE 4

See MINITAB steps on page 424.

Testing μ with a Small Sample

A used car dealer says that the mean price of a 2008 Honda CR-V is at least \$20,500. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$19,850 and a standard deviation of \$1084. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed. *(Adapted from Kelley Blue Book)*

Solution

The claim is "the mean price is at least \$20,500." So, the null and alternative hypotheses are

H_0: $\mu \geq \$20{,}500$ (Claim)

and

H_a: $\mu < \$20{,}500$.

The test is a left-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are d.f. $= 14 - 1 = 13$. So, the critical value is $t_0 = -1.771$. The rejection region is $t < -1.771$. The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad \text{Because } n < 30\text{, use the } t\text{-test.}$$

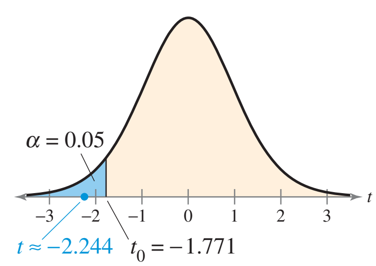
$$= \frac{19{,}850 - 20{,}500}{1084/\sqrt{14}} \qquad \text{Assume } \mu = 20{,}500.$$

$$\approx -2.244.$$

To explore this topic further, see Activity 7.3 on page 397.

The graph shows the location of the rejection region and the standardized test statistic t. Because t is in the rejection region, you should reject the null hypothesis.

Interpretation There is enough evidence at the 5% level of significance to reject the claim that the mean price of a 2008 Honda CR-V is at least \$20,500.



5% Level of Significance

Try It Yourself 4

An insurance agent says that the mean cost of insuring a 2008 Honda CR-V is less than \$1200. A random sample of 7 similar insurance quotes has a mean cost of \$1125 and a standard deviation of \$55. Is there enough evidence to support the agent's claim at $\alpha = 0.10$? Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α and the *degrees of freedom*.
c. Find the *critical value* t_0 and identify the *rejection region*.
d. Find the *standardized test statistic* t. *Sketch* a graph.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A41

EXAMPLE 5

See TI-83/84 Plus steps on page 425.

Testing μ with a Small Sample

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 19 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.24, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$? Assume the population is normally distributed.

Solution

The claim is "the mean pH level is 6.8." So, the null and alternative hypotheses are

H_0: $\mu = 6.8$ (Claim)

and

H_a: $\mu \neq 6.8$.

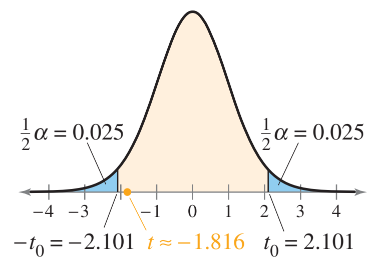
The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are d.f. $= 19 - 1 = 18$. So, the critical values are $-t_0 = -2.101$ and $t_0 = 2.101$. The rejection regions are $t < -2.101$ and $t > 2.101$. The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad \text{Because } n < 30\text{, use the } t\text{-test.}$$

$$= \frac{6.7 - 6.8}{0.24/\sqrt{19}} \qquad \text{Assume } \mu = 6.8.$$

$$\approx -1.816.$$

The graph shows the location of the rejection region and the standardized test statistic t. Because t is not in the rejection region, you fail to reject the null hypothesis.



5% Level of Significance

Interpretation There is not enough evidence at the 5% level of significance to reject the claim that the mean pH is 6.8.

Try It Yourself 5

The company also claims that the mean conductivity of the river is 1890 milligrams per liter. The conductivity of a water sample is a measure of the total dissolved solids in the sample. You randomly select 19 water samples and measure the conductivity of each. The sample mean and standard deviation are 2500 milligrams per liter and 700 milligrams per liter, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.01$? Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α and the *degrees of freedom*.
c. Find the *critical values* $-t_0$ and t_0 and identify the *rejection region*.
d. Find the *standardized test statistic* t. *Sketch* a graph.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A42

USING *P*-VALUES WITH *t*-TESTS

Suppose you wanted to find a *P*-value given $t = 1.98$, 15 degrees of freedom, and a right-tailed test. Using Table 5 in Appendix B, you can determine that *P* falls between $\alpha = 0.025$ and $\alpha = 0.05$, but you cannot determine an exact value for *P*. In such cases, you can use technology to perform a hypothesis test and find exact *P*-values.

STUDY TIP

Using a TI-83/84 Plus, you can either enter the original data into a list to find a *P*-value or enter the descriptive statistics.

STAT

Choose the TESTS menu.

2: T-Test…

Select the *Data* input option if you use the original data. Select the *Stats* input option if you use the descriptive statistics. In each case, enter the appropriate values including the corresponding type of hypothesis test indicated by the alternative hypothesis. Then select *Calculate*.

EXAMPLE 6

SC Report 30

Using *P*-Values with a *t*-Test

A Department of Motor Vehicles office claims that the mean wait time is less than 14 minutes. A random sample of 10 people has a mean wait time of 13 minutes with a standard deviation of 3.5 minutes. At $\alpha = 0.10$, test the office's claim. Assume the population is normally distributed.

Solution

The claim is "the mean wait time is less than 14 minutes." So, the null and alternative hypotheses are

H_0: $\mu \geq 14$ minutes

and

H_a: $\mu < 14$ minutes. (Claim)

The TI-83/84 Plus display at the far left shows how to set up the hypothesis test. The two displays on the right show the possible results, depending on whether you select "Calculate" or "Draw."

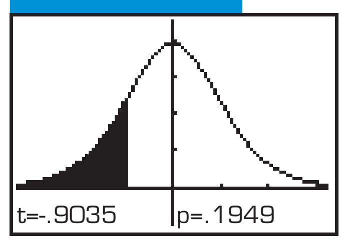
TI-83/84 PLUS

```
T-Test
  Inpt: Data Stats
  μ0:14
  x̄:13
  Sx:3.5
  n:10
  μ:≠μ0 <μ0 >μ0
  Calculate Draw
```

TI-83/84 PLUS

```
T-Test
  μ<14
  t=-.9035079029
  p=.1948994027
  x̄=13
  Sx=3.5
  n=10
```

TI-83/84 PLUS



From the displays, you can see that $P \approx 0.1949$. Because the *P*-value is greater than $\alpha = 0.10$, you fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 10% level of significance to support the office's claim that the mean wait time is less than 14 minutes.

Try It Yourself 6

Another Department of Motor Vehicles office claims that the mean wait time is at most 18 minutes. A random sample of 12 people has a mean wait time of 15 minutes with a standard deviation of 2.2 minutes. At $\alpha = 0.05$, test the office's claim. Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. *Use* a TI-83/84 Plus to find the *P*-value.
c. *Compare* the *P*-value with the level of significance α and *make* a decision.
d. *Interpret* the decision in the context of the original claim.

Answer: Page A42

7.3 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. Explain how to find critical values for a t-sampling distribution.

2. Explain how to use a t-test to test a hypothesized mean μ given a small sample $(n < 30)$. What assumption about the population is necessary?

In Exercises 3–8, find the critical value(s) for the indicated t-test, level of significance α, and sample size n.

3. Right-tailed test, $\alpha = 0.05$, $n = 23$

4. Right-tailed test, $\alpha = 0.01$, $n = 11$

5. Left-tailed test, $\alpha = 0.10$, $n = 20$

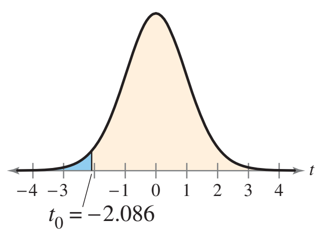
6. Left-tailed test, $\alpha = 0.01$, $n = 28$

7. Two-tailed test, $\alpha = 0.05$, $n = 27$

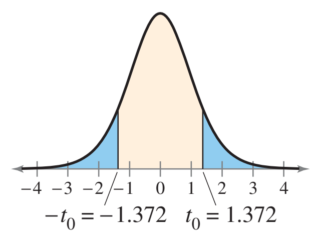
8. Two-tailed test, $\alpha = 0.10$, $n = 22$

Graphical Analysis *In Exercises 9–12, state whether the standardized test statistic t indicates that you should reject the null hypothesis. Explain.*

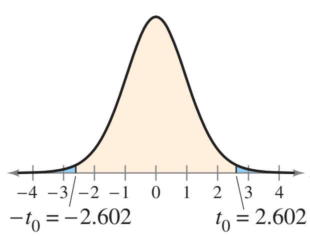
9. (a) $t = 2.091$
(b) $t = 0$
(c) $t = -1.08$
(d) $t = -2.096$



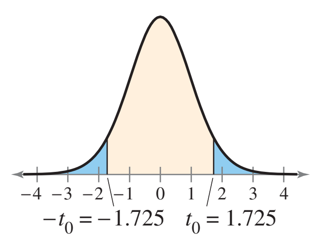
10. (a) $t = 1.308$
(b) $t = -1.389$
(c) $t = 1.650$
(d) $t = -0.998$



11. (a) $t = -2.502$
(b) $t = 2.203$
(c) $t = 2.680$
(d) $t = -2.703$



12. (a) $t = 1.705$
(b) $t = -1.755$
(c) $t = -1.585$
(d) $t = 1.745$



In Exercises 13–16, use a t-test to test the claim about the population mean μ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed.

13. Claim: $\mu = 15$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 13.9$, $s = 3.23$, $n = 6$

14. Claim: $\mu > 25$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 26.2$, $s = 2.32$, $n = 17$

15. Claim: $\mu \geq 8000$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 7700$, $s = 450$, $n = 25$

16. Claim: $\mu \neq 52{,}200$; $\alpha = 0.10$. Sample statistics: $\bar{x} = 53{,}220$, $s = 2700$, $n = 18$

USING AND INTERPRETING CONCEPTS

Testing Claims *In Exercises 17–24, (a) write the claim mathematically and identify H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic t, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. If convenient, use technology. For each claim, assume the population is normally distributed.*

17. **Used Car Cost** A used car dealer says that the mean price of a 2008 Subaru Forester is \$18,000. You suspect this claim is incorrect and find that a random sample of 15 similar vehicles has a mean price of \$18,550 and a standard deviation of \$1767. Is there enough evidence to reject the claim at $\alpha = 0.05$? *(Adapted from Kelley Blue Book)*

18. **IRS Wait Times** The Internal Revenue Service claims that the mean wait time for callers during a recent tax filing season was at most 7 minutes. A random sample of 11 callers has a mean wait time of 8.7 minutes and a standard deviation of 2.7 minutes. Is there enough evidence to reject the claim at $\alpha = 0.10$? *(Adapted from Internal Revenue Service)*

19. **Work Hours** A medical board claims that the mean number of hours worked per week by surgical faculty who teach at an academic institution is more than 60 hours. The hours worked include teaching hours as well as regular working hours. A random sample of 7 surgical faculty has a mean hours worked per week of 70 hours and a standard deviation of 12.5 hours. At $\alpha = 0.05$, do you have enough evidence to support the board's claim? *(Adapted from Journal of the American College of Surgeons)*

20. **Battery Life** A company claims that the mean battery life of their MP3 player is at least 30 hours. You suspect this claim is incorrect and find that a random sample of 18 MP3 players has a mean battery life of 28.5 hours and a standard deviation of 1.7 hours. Is there enough evidence to reject the claim at $\alpha = 0.01$?

21. **Waste Recycled** An environmentalist estimates that the mean amount of waste recycled by adults in the United States is more than 1 pound per person per day. You want to test this claim. You find that the mean waste recycled per person per day for a random sample of 13 adults in the United States is 1.50 pounds and the standard deviation is 0.28 pound. At $\alpha = 0.10$, can you support the claim? *(Adapted from U.S. Environmental Protection Agency)*

22. **Waste Generated** As part of your work for an environmental awareness group, you want to test a claim that the mean amount of waste generated by adults in the United States is more than 4 pounds per day. In a random sample of 22 adults in the United States, you find that the mean waste generated per person per day is 4.50 pounds with a standard deviation of 1.21 pounds. At $\alpha = 0.01$, can you support the claim? *(Adapted from U.S. Environmental Protection Agency)*

23. **Annual Pay** An employment information service claims the mean annual salary for full-time male workers over age 25 and without a high school diploma is \$26,000. The annual salaries for a random sample of 10 full-time male workers without a high school diploma are listed. At $\alpha = 0.05$, test the claim that the mean salary is \$26,000. *(Adapted from U.S. Bureau of Labor Statistics)*

26,185	23,814	22,374	25,189	26,318
20,767	30,782	29,541	24,597	28,955

24. Annual Pay An employment information service claims the mean annual salary for full-time female workers over age 25 and without a high school diploma is more than \$18,500. The annual salaries for a random sample of 12 full-time female workers without a high school diploma are listed. At $\alpha = 0.10$, is there enough evidence to support the claim that the mean salary is more than \$18,500? *(Adapted from U.S. Bureau of Labor Statistics)*

18,665	16,312	18,794	19,403	20,864	19,177
17,328	21,445	20,354	19,143	18,316	19,237

Testing Claims Using *P*-Values *In Exercises 25–30, (a) write the claim mathematically and identify H_0 and H_a, (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim. Assume the population is normally distributed.*

25. Speed Limit A county is considering raising the speed limit on a road because they claim that the mean speed of vehicles is greater than 45 miles per hour. A random sample of 25 vehicles has a mean speed of 48 miles per hour and a standard deviation of 5.4 miles per hour. At $\alpha = 0.10$, do you have enough evidence to support the county's claim?

26. Oil Changes A repair shop believes that people travel more than 3500 miles between oil changes. A random sample of 8 cars getting an oil change has a mean distance of 3375 miles since having an oil change with a standard deviation of 225 miles. At $\alpha = 0.05$, do you have enough evidence to support the shop's claim?

27. Meal Cost A travel association claims that the mean daily meal cost for two adults traveling together on vacation in San Francisco is \$105. A random sample of 20 such groups of adults has a mean daily meal cost of \$110 and a standard deviation of \$8.50. Is there enough evidence to reject the claim at $\alpha = 0.01$? *(Adapted from American Automobile Association)*

28. Lodging Cost A travel association claims that the mean daily lodging cost for two adults traveling together on vacation in San Francisco is at least \$240. A random sample of 24 such groups of adults has a mean daily lodging cost of \$233 and a standard deviation of \$12.50. Is there enough evidence to reject the claim at $\alpha = 0.10$? *(Adapted from American Automobile Association)*

29. Class Size You receive a brochure from a large university. The brochure indicates that the mean class size for full-time faculty is fewer than 32 students. You want to test this claim. You randomly select 18 classes taught by full-time faculty and determine the class size of each. The results are listed below. At $\alpha = 0.05$, can you support the university's claim?

35	28	29	33	32	40	26	25	29
28	30	36	33	29	27	30	28	25

30. Faculty Classroom Hours The dean of a university estimates that the mean number of classroom hours per week for full-time faculty is 11.0. As a member of the student council, you want to test this claim. A random sample of the number of classroom hours for eight full-time faculty for one week is listed below. At $\alpha = 0.01$, can you reject the dean's claim?

11.8	8.6	12.6	7.9	6.4	10.4	13.6	9.1

SC *In Exercises 31–34, use StatCrunch and a t-test to help you test the claim about the population mean μ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed.*

31. Claim: $\mu \leq 75$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 73.6$, $s = 3.2$, $n = 26$

32. Claim: $\mu \neq 27$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 31.5$, $s = 4.7$, $n = 12$

33. Claim: $\mu < 188$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 186$, $s = 12$, $n = 9$

34. Claim: $\mu \geq 2118$; $\alpha = 0.10$. Sample statistics: $\bar{x} = 1787$, $s = 384$, $n = 17$

EXTENDING CONCEPTS

35. Credit Card Balances To test the claim that the mean credit card debt for individuals is greater than \$5000, you do some research and find that a random sample of 6 cardholders has a mean credit card balance of \$5434 with a standard deviation of \$625. You conduct a statistical experiment where H_0: $\mu \leq \$5000$ and H_a: $\mu > \$5000$. At $\alpha = 0.05$, explain why you cannot reject H_0. Assume the population is normally distributed. *(Adapted from TransUnion)*

36. Using Different Values of α and n In Exercise 35, you believe that H_0 is not valid. Which of the following allows you to reject H_0? Explain your reasoning.

(a) Use the same values but decrease α from 0.05 to 0.01.

(b) Use the same values but increase α from 0.05 to 0.10.

(c) Use the same values but increase n from 6 to 8.

(d) Use the same values but increase n from 6 to 24.

Deciding on a Distribution *In Exercises 37 and 38, decide whether you should use a normal sampling distribution or a t-sampling distribution to perform the hypothesis test. Justify your decision. Then use the distribution to test the claim. Write a short paragraph about the results of the test and what you can conclude about the claim.*

37. Gas Mileage A car company says that the mean gas mileage for its luxury sedan is at least 23 miles per gallon (mpg). You believe the claim is incorrect and find that a random sample of 5 cars has a mean gas mileage of 22 mpg and a standard deviation of 4 mpg. At $\alpha = 0.05$, test the company's claim. Assume the population is normally distributed.

38. Private Law School An education publication claims that the average in-state tuition for one year of law school at a private institution is more than \$35,000. A random sample of 50 private law schools has a mean in-state tuition of \$34,967 and a standard deviation of \$5933 for one year. At $\alpha = 0.01$, test the publication's claim. Assume the population is normally distributed. *(Adapted from U.S. News and World Report)*

39. Writing You are testing a claim and incorrectly use the normal sampling distribution instead of the t-sampling distribution. Does this make it more or less likely to reject the null hypothesis? Is this result the same no matter whether the test is left-tailed, right-tailed, or two-tailed? Explain your reasoning.

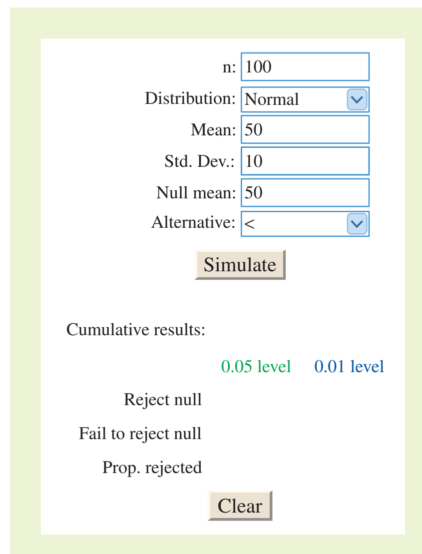
ACTIVITY 7.3 Hypothesis Tests for a Mean

APPLET

The *hypothesis tests for a mean* applet allows you to visually investigate hypothesis tests for a mean. You can specify the sample size n, the shape of the distribution (Normal or Right skewed), the true population mean (Mean), the true population standard deviation (Std. Dev.), the null value for the mean (Null mean), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size n will be selected from a population with these population parameters. For each of the 100 samples, a hypothesis test based on the T statistic is performed, and the results from each test are displayed in the plots at the right. The test statistic for each test is shown in the top plot and the P-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.

Explore

Step 1 Specify a value for n.
Step 2 Specify a distribution.
Step 3 Specify a value for the mean.
Step 4 Specify a value for the standard deviation.
Step 5 Specify a value for the null mean.
Step 6 Specify an alternative hypothesis.
Step 7 Click SIMULATE to generate the hypothesis tests.



Draw Conclusions

APPLET

1. Set $n = 15$, Mean $= 40$, Std. Dev. $= 5$, Null mean $= 40$, alternative hypothesis to “not equal,” and the distribution to “Normal.” Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

2. Suppose a null hypothesis is rejected at the 0.01 level. Will it be rejected at the 0.05 level? Explain. Suppose a null hypothesis is rejected at the 0.05 level. Will it be rejected at the 0.01 level? Explain.

3. Set $n = 25$, Mean $= 25$, Std. Dev. $= 3$, Null mean $= 27$, alternative hypothesis to “$<$,” and the distribution to “Normal.” What is the null hypothesis? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

7.4 Hypothesis Testing for Proportions

WHAT YOU SHOULD LEARN

- How to use the z-test to test a population proportion p

Hypothesis Test for Proportions

HYPOTHESIS TEST FOR PROPORTIONS

In Sections 7.2 and 7.3, you learned how to perform a hypothesis test for a population mean. In this section, you will learn how to test a population proportion p.

Hypothesis tests for proportions can be used when politicians want to know the proportion of their constituents who favor a certain bill or when quality assurance engineers test the proportion of parts that are defective.

If $np \geq 5$ and $nq \geq 5$ for a binomial distribution, then the sampling distribution for $\hat{p}$ is approximately normal with a mean of

$$\mu_{\hat{p}} = p$$

and a standard error of

$$\sigma_{\hat{p}} = \sqrt{pq/n}.$$

z-TEST FOR A PROPORTION *p*

The ***z*-test for a proportion** is a statistical test for a population proportion p. The z-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$. The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

GUIDELINES

Using a *z*-Test for a Proportion *p*

Verify that $np \geq 5$ and $nq \geq 5$.

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the critical value(s).	Use Table 4 in Appendix B.
4. Determine the rejection region(s).	
5. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$
6. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.	

INSIGHT

A hypothesis test for a proportion p can also be performed using P-values. Use the guidelines on page 373 for using P-values for a z-test for a mean μ, but in Step 3 find the standardized test statistic by using the formula

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

The other steps in the test are the same.

EXAMPLE 1

See TI-83/84 Plus steps on page 425.

Hypothesis Test for a Proportion

A research center claims that less than 50% of U.S. adults have accessed the Internet over a wireless network with a laptop computer. In a random sample of 100 adults, 39% say they have accessed the Internet over a wireless network with a laptop computer. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim? *(Adapted from Pew Research Center)*

Solution The products $np = 100(0.50) = 50$ and $nq = 100(0.50) = 50$ are both greater than 5. So, you can use a z-test. The claim is "less than 50% have accessed the Internet over a wireless network with a laptop computer." So, the null and alternative hypotheses are

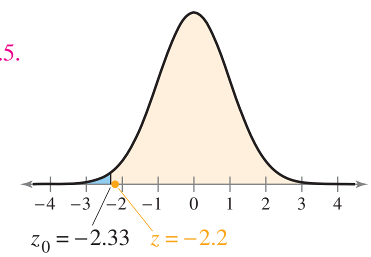
$$H_0: p \geq 0.5 \quad \text{and} \quad H_a: p < 0.5. \text{ (Claim)}$$

Because the test is a left-tailed test and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$ and the rejection region is $z < -2.33$. The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \qquad \text{Because } np \geq 5 \text{ and } nq \geq 5\text{, you can use the } z\text{-test.}$$

$$= \frac{0.39 - 0.5}{\sqrt{(0.5)(0.5)/100}} \qquad \text{Assume } p = 0.5.$$

$$= -2.2.$$



1% Level of Significance

The graph shows the location of the rejection region and the standardized test statistic z. Because z is not in the rejection region, you should fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 1% level of significance to support the claim that less than 50% of U.S. adults have accessed the Internet over a wireless network with a laptop computer.

Try It Yourself 1

A research center claims that more than 25% of U.S. adults have used a cellular phone to access the Internet. In a random sample of 125 adults, 32% say they have used a cellular phone to access the Internet. At $\alpha = 0.05$, is there enough evidence to support the researcher's claim? *(Adapted from Pew Research Center)*

a. *Verify* that $np \geq 5$ and $nq \geq 5$.
b. Identify the *claim* and state H_0 and H_a.
c. Identify the *level of significance* α.
d. Find the *critical value* z_0 and identify the *rejection region*.
e. Find the *standardized test statistic* z. *Sketch* a graph.
f. *Decide* whether to reject the null hypothesis.
g. *Interpret* the decision in the context of the original claim.

Answer: Page A42

To use a P-value to perform the hypothesis test in Example 1, use Table 4 to find the area corresponding to $z = -2.2$. The area is 0.0139. Because this is a left-tailed test, the P-value is equal to the area to the left of $z = -2.2$. So, $P = 0.0139$. Because the P-value is greater than $\alpha = 0.01$, you should fail to reject the null hypothesis. Note that this is the same result obtained in Example 1.

To explore this topic further, see Activity 7.4 on page 403.

STUDY TIP

Remember that if the sample proportion is not given, you can find it using

$$\hat{p} = \frac{x}{n}$$

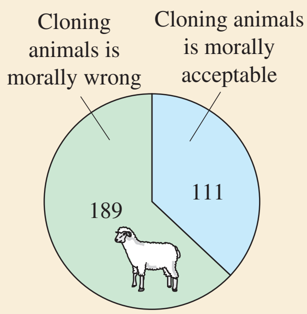
where x is the number of successes in the sample and n is the sample size.

STUDY TIP

Remember that when you fail to reject H_0, a type II error is possible. For instance, in Example 1 the null hypothesis, $p \geq 0.5$, may be false.

PICTURING THE WORLD

A recent survey claimed that at least 70% of U.S. adults believe that cloning animals is morally wrong. To test this claim, you conduct a random telephone survey of 300 U.S. adults. In the survey, you find that 189 adults believe that cloning animals is morally wrong. (Adapted from The Gallup Poll)



At $\alpha = 0.05$, is there enough evidence to reject the claim?

EXAMPLE 2

See MINITAB steps on page 424.

Hypothesis Test for a Proportion

A research center claims that 25% of college graduates think a college degree is not worth the cost. You decide to test this claim and ask a random sample of 200 college graduates whether they think a college degree is not worth the cost. Of those surveyed, 21% reply yes. At $\alpha = 0.10$, is there enough evidence to reject the claim? *(Adapted from Zogby International)*

Solution

The products $np = 200(0.25) = 50$ and $nq = 200(0.75) = 150$ are both greater than 5. So, you can use a z-test. The claim is "25% of college graduates think a college degree is not worth the cost." So, the null and alternative hypotheses are

$$H_0: p = 0.25 \text{ (Claim)} \quad \text{and} \quad H_a: p \neq 0.25.$$

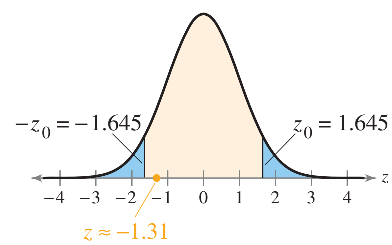
Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$. The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \qquad \text{Because } np \geq 5 \text{ and } nq \geq 5\text{, you can use the } z\text{-test.}$$

$$= \frac{0.21 - 0.25}{\sqrt{(0.25)(0.75)/200}} \qquad \text{Assume } p = 0.25.$$

$$= -1.31.$$

The graph shows the location of the rejection regions and the standardized test statistic z. Because z is not in the rejection region, you should fail to reject the null hypothesis.



10% Level of Significance

Interpretation There is not enough evidence at the 10% level of significance to reject the claim that 25% of college graduates think a college degree is not worth the cost.

Try It Yourself 2

A research center claims that 30% of U.S. adults have not purchased a certain brand because they found the advertisements distasteful. You decide to test this claim and ask a random sample of 250 U.S. adults whether they have not purchased a certain brand because they found the advertisements distasteful. Of those surveyed, 36% reply yes. At $\alpha = 0.10$, is there enough evidence to reject the claim? *(Adapted from Harris Interactive)*

a. *Verify* that $np \geq 5$ and $nq \geq 5$.
b. Identify the *claim* and state H_0 and H_a.
c. Identify the *level of significance* α.
d. Find the *critical values* $-z_0$ and z_0 and identify the *rejection regions*.
e. Find the *standardized test statistic* z. *Sketch* a graph.
f. *Decide* whether to reject the null hypothesis.
g. *Interpret* the decision in the context of the original claim.

Answer: Page A42

7.4 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. Explain how to decide when a normal distribution can be used to approximate a binomial distribution.

2. Explain how to test a population proportion p.

In Exercises 3–8, decide whether the normal sampling distribution can be used. If it can be used, test the claim about the population proportion p at the given level of significance α using the given sample statistics.

3. Claim: $p < 0.12$; $\alpha = 0.01$. Sample statistics: $\hat{p} = 0.10$, $n = 40$

4. Claim: $p \geq 0.48$; $\alpha = 0.08$. Sample statistics: $\hat{p} = 0.40$, $n = 90$

5. Claim: $p \neq 0.15$; $\alpha = 0.05$. Sample statistics: $\hat{p} = 0.12$, $n = 500$

6. Claim: $p > 0.70$; $\alpha = 0.04$. Sample statistics: $\hat{p} = 0.64$, $n = 225$

7. Claim: $p \leq 0.45$; $\alpha = 0.05$. Sample statistics: $\hat{p} = 0.52$, $n = 100$

8. Claim: $p = 0.95$; $\alpha = 0.10$. Sample statistics: $\hat{p} = 0.875$, $n = 50$

USING AND INTERPRETING CONCEPTS

Testing Claims *In Exercises 9–16, (a) write the claim mathematically and identify H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic z, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. If convenient, use technology to find the standardized test statistic.*

9. Smokers A medical researcher says that less than 25% of U.S. adults are smokers. In a random sample of 200 U.S. adults, 18.5% say that they are smokers. At $\alpha = 0.05$, is there enough evidence to reject the researcher's claim? *(Adapted from National Center for Health Statistics)*

10. Census A research center claims that at least 40% of U.S. adults think the Census count is accurate. In a random sample of 600 U.S. adults, 35% say that the Census count is accurate. At $\alpha = 0.02$, is there enough evidence to reject the center's claim? *(Adapted from Rasmussen Reports)*

11. Cellular Phones and Driving A research center claims that at most 50% of people believe that drivers should be allowed to use cellular phones with hands-free devices while driving. In a random sample of 150 U.S. adults, 58% say that drivers should be allowed to use cellular phones with hands-free devices while driving. At $\alpha = 0.01$, is there enough evidence to reject the center's claim? *(Adapted from Rasmussen Reports)*

12. Asthma A medical researcher claims that 5% of children under 18 years of age have asthma. In a random sample of 250 children under 18 years of age, 9.6% say they have asthma. At $\alpha = 0.08$, is there enough evidence to reject the researcher's claim? *(Adapted from National Center for Health Statistics)*

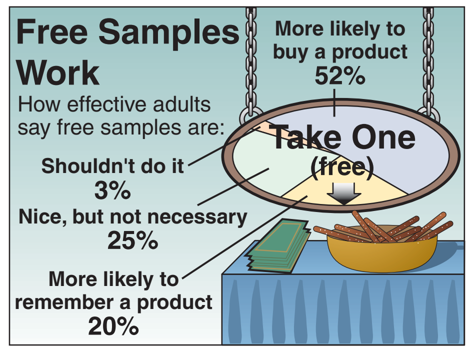
13. Female Height A research center claims that more than 75% of females ages 20–29 are taller than 62 inches. In a random sample of 150 females ages 20–29, 82% are taller than 62 inches. At $\alpha = 0.10$, is there enough evidence to support the center's claim? *(Adapted from National Center for Health Statistics)*

14. Curling A research center claims that 16% of U.S. adults say that curling is the Winter Olympic sport they would like to try the most. In a random sample of 300 U.S. adults, 20% say that curling is the Winter Olympic sport they would like to try the most. At $\alpha = 0.05$, is there enough evidence to reject the researcher's claim? *(Adapted from Zogby International)*

15. Dog Ownership A humane society claims that less than 35% of U.S. households own a dog. In a random sample of 400 U.S. households, 156 say they own a dog. At $\alpha = 0.10$, is there enough evidence to support the society's claim? *(Adapted from The Humane Society of the United States)*

16. Cat Ownership A humane society claims that 30% of U.S. households own a cat. In a random sample of 200 U.S. households, 72 say they own a cat. At $\alpha = 0.05$, is there enough evidence to reject the society's claim? *(Adapted from The Humane Society of the United States)*

Free Samples *In Exercises 17 and 18, use the graph, which shows what adults think about the effectiveness of free samples.*



17. Do Free Samples Work? You interview a random sample of 50 adults. The results of the survey show that 48% of the adults said they were more likely to buy a product when there are free samples. At $\alpha = 0.05$, can you reject the claim that at least 52% of adults are more likely to buy a product when there are free samples?

18. Should Free Samples Be Used? Use your conclusion from Exercise 17 to write a paragraph on the use of free samples. Do you think a company should use free samples to get people to buy a product? Explain.

EXTENDING CONCEPTS

Alternative Formula *In Exercises 19 and 20, use the following information. When you know the number of successes x, the sample size n, and the population proportion p, it can be easier to use the formula*

$$z = \frac{x - np}{\sqrt{npq}}$$

to find the standardized test statistic when using a z-test for a population proportion p.

19. Rework Exercise 15 using the alternative formula and compare the results.

20. The alternative formula is derived from the formula

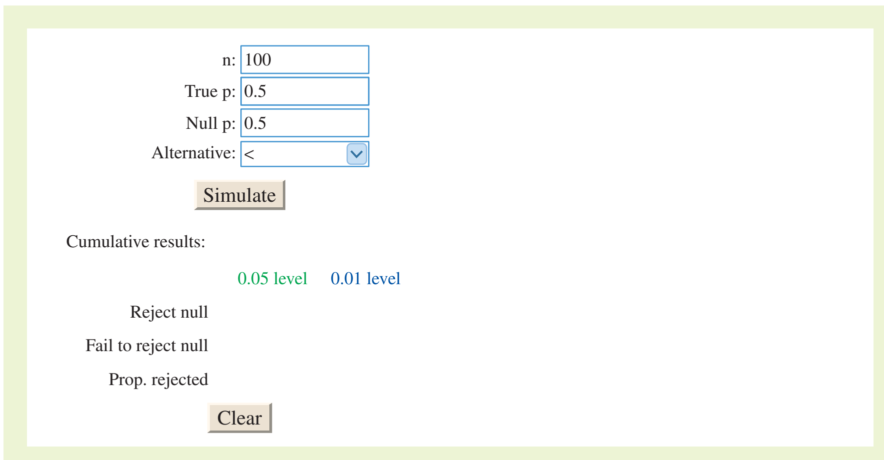
$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{(x/n) - p}{\sqrt{pq/n}}.$$

Use this formula to derive the alternative formula. Justify each step.

ACTIVITY 7.4 Hypothesis Tests for a Proportion

APPLET

The *hypothesis tests for a proportion* applet allows you to visually investigate hypothesis tests for a population proportion. You can specify the sample size n, the population proportion (True p), the null value for the proportion (Null p), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size n will be selected from a population with a proportion of successes equal to True p. For each of the 100 samples, a hypothesis test based on the Z statistic is performed, and the results from each test are displayed in plots at the right. The standardized test statistic for each test is shown in the top plot and the P-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.



Explore

Step 1 Specify a value for n.
Step 2 Specify a value for True p.
Step 3 Specify a value for Null p.
Step 4 Specify an alternative hypothesis.
Step 5 Click SIMULATE to generate the hypothesis tests.

Draw Conclusions

APPLET

1. Set $n = 25$, True $p = 0.35$, Null $p = 0.35$, and the alternative hypothesis to "not equal." Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

2. Set $n = 50$, True $p = 0.6$, Null $p = 0.4$, and the alternative hypothesis to "$<$." What is the null hypothesis? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Perform a hypothesis test for each level. Use the results of the hypothesis tests to explain the results of the simulation.

7.5 Hypothesis Testing for Variance and Standard Deviation

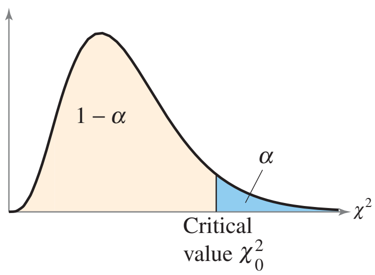
WHAT YOU SHOULD LEARN

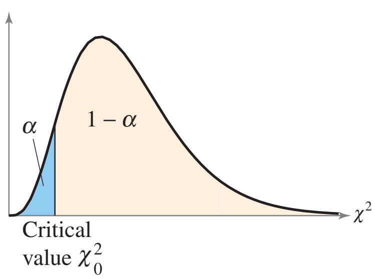
- How to find critical values for a χ^2-test
- How to use the χ^2-test to test a variance or a standard deviation

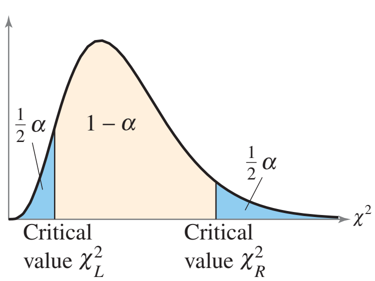
Critical Values for a χ^2-Test ▸ The Chi-Square Test

▸ CRITICAL VALUES FOR A χ^2-TEST

In real life, it is often important to produce consistent predictable results. For instance, consider a company that manufactures golf balls. The manufacturer must produce millions of golf balls, each having the same size and the same weight. There is a very low tolerance for variation. If the population is normal, you can test the variance and standard deviation of the process using the chi-square distribution with $n - 1$ degrees of freedom.







GUIDELINES

Finding Critical Values for the χ^2-Test

1. Specify the level of significance α.
2. Determine the degrees of freedom d.f. $= n - 1$.
3. The critical values for the χ^2-distribution are found in Table 6 in Appendix B. To find the critical value(s) for a
   - **a.** *right-tailed test*, use the value that corresponds to d.f. and α.
   - **b.** *left-tailed test*, use the value that corresponds to d.f. and $1 - \alpha$.
   - **c.** *two-tailed test*, use the values that correspond to d.f. and $\frac{1}{2}\alpha$, and d.f. and $1 - \frac{1}{2}\alpha$.

EXAMPLE 1

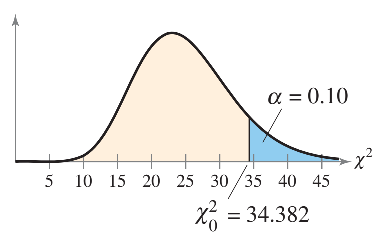
▸ Finding Critical Values for χ^2

Find the critical χ^2-value for a right-tailed test when $n = 26$ and $\alpha = 0.10$.

▸ Solution

The degrees of freedom are

$$\text{d.f.} = n - 1 = 26 - 1 = 25.$$



The graph at the right shows a χ^2-distribution with 25 degrees of freedom and a shaded area of $\alpha = 0.10$ in the right tail. In Table 6 in Appendix B with d.f. $= 25$ and $\alpha = 0.10$, the critical value is

$$\chi_0^2 = 34.382.$$

▸ Try It Yourself 1

Find the critical χ^2-value for a right-tailed test when $n = 18$ and $\alpha = 0.01$.

a. *Identify* the degrees of freedom and the level of significance.
b. *Use* Table 6 in Appendix B to find the critical χ^2-value.

Answer: Page A42

EXAMPLE 2

Finding Critical Values for χ^2

Find the critical χ^2-value for a left-tailed test when $n = 11$ and $\alpha = 0.01$.

Solution

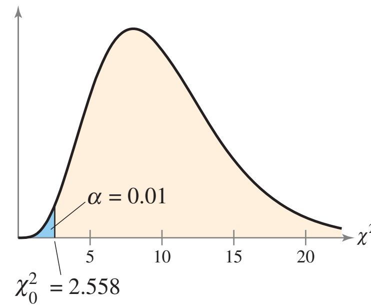
The degrees of freedom are

$$\text{d.f.} = n - 1 = 11 - 1 = 10.$$

The graph shows a χ^2-distribution with 10 degrees of freedom and a shaded area of $\alpha = 0.01$ in the left tail. The area to the right of the critical value is

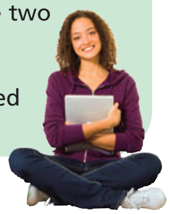
$$1 - \alpha = 1 - 0.01 = 0.99.$$

In Table 6 with d.f. $= 10$ and the area $1 - \alpha = 0.99$, the critical value is $\chi_0^2 = 2.558$.



STUDY TIP

Note that because chi-square distributions are not symmetric (like normal or t-distributions), in a two-tailed test the two critical values are not opposites. Each critical value must be calculated separately.



Try It Yourself 2

Find the critical χ^2-value for a left-tailed test when $n = 30$ and $\alpha = 0.05$.

a. *Identify* the degrees of freedom and the level of significance.
b. *Use* Table 6 in Appendix B to find the critical χ^2-value.

Answer: Page A42

EXAMPLE 3

Finding Critical Values for χ^2

Find the critical χ^2-values for a two-tailed test when $n = 9$ and $\alpha = 0.05$.

Solution

The degrees of freedom are

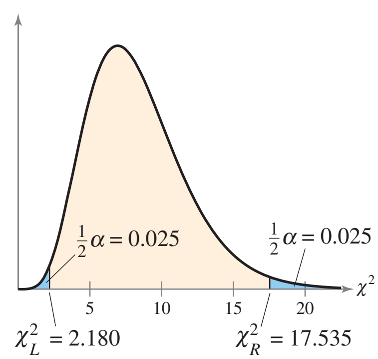
$$\text{d.f.} = n - 1 = 9 - 1 = 8.$$

The graph shows a χ^2-distribution with 8 degrees of freedom and a shaded area of $\frac{1}{2}\alpha = 0.025$ in each tail. The areas to the right of the critical values are

$$\tfrac{1}{2}\alpha = 0.025$$

and

$$1 - \tfrac{1}{2}\alpha = 0.975.$$



In Table 6 with d.f. $= 8$ and the areas 0.025 and 0.975, the critical values are $\chi_L^2 = 2.180$ and $\chi_R^2 = 17.535$.

Try It Yourself 3

Find the critical χ^2-values for a two-tailed test when $n = 51$ and $\alpha = 0.01$.

a. *Identify* the degrees of freedom and the level of significance.
b. *Find* the first critical value χ_R^2 using Table 6 in Appendix B and the area $\frac{1}{2}\alpha$.
c. *Find* the second critical value χ_L^2 using Table 6 in Appendix B and the area $1 - \frac{1}{2}\alpha$.

Answer: Page A42

▸ THE CHI-SQUARE TEST

To test a variance σ^2 or a standard deviation σ of a population that is normally distributed, you can use the χ^2-test. The χ^2-test for a variance or standard deviation is not as robust as the tests for the population mean μ or the population proportion p. So, it is essential in performing a χ^2-test for a variance or standard deviation that the population be normally distributed. The results can be misleading if the population is not normal.

χ^2-TEST FOR A VARIANCE σ^2 OR STANDARD DEVIATION σ

The **χ^2-test for a variance or standard deviation** is a statistical test for a population variance or standard deviation. The χ^2-test can be used when the population is normal. The **test statistic** is s^2 and the **standardized test statistic**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

follows a chi-square distribution with degrees of freedom

d.f. $= n - 1$.

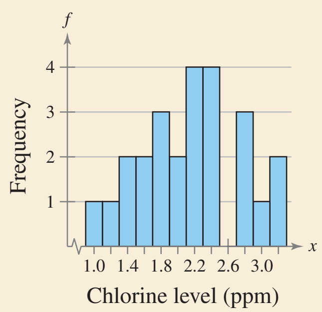
GUIDELINES

Using the χ^2-Test for a Variance or Standard Deviation

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the degrees of freedom.	d.f. $= n - 1$
4. Determine the critical value(s).	Use Table 6 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

PICTURING THE WORLD

A community center claims that the chlorine level in its pool has a standard deviation of 0.46 parts per million (ppm). A sampling of the pool's chlorine levels at 25 random times during a month yields a standard deviation of 0.61 ppm. (Adapted from American Pool Supply)



At $\alpha = 0.05$, is there enough evidence to reject the claim?

EXAMPLE 4

SC Report 31

Using a Hypothesis Test for the Population Variance

A dairy processing company claims that the variance of the amount of fat in the whole milk processed by the company is no more than 0.25. You suspect this is wrong and find that a random sample of 41 milk containers has a variance of 0.27. At $\alpha = 0.05$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

Solution

The claim is "the variance is no more than 0.25." So, the null and alternative hypotheses are

$$H_0: \sigma^2 \le 0.25 \text{ (Claim)} \qquad \text{and} \qquad H_a: \sigma^2 > 0.25.$$

The test is a right-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are d.f. $= 41 - 1 = 40$. So, the critical value is

$$\chi_0^2 = 55.758.$$

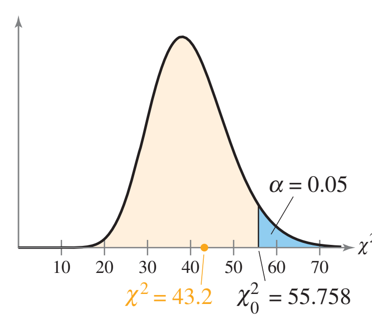
The rejection region is $\chi^2 > 55.758$. The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \text{Use the chi-square test.}$$

$$= \frac{(41-1)(0.27)}{0.25} \qquad \text{Assume } \sigma^2 = 0.25.$$

$$= 43.2.$$

The graph shows the location of the rejection region and the standardized test statistic χ^2. Because χ^2 is not in the rejection region, you should fail to reject the null hypothesis.



Interpretation There is not enough evidence at the 5% level of significance to reject the company's claim that the variance of the amount of fat in the whole milk is no more than 0.25.

Try It Yourself 4

A bottling company claims that the variance of the amount of sports drink in a 12-ounce bottle is no more than 0.40. A random sample of 31 bottles has a variance of 0.75. At $\alpha = 0.01$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α and the *degrees of freedom*.
c. Find the *critical value* and identify the *rejection region*.
d. Find the *standardized test statistic* χ^2.
e. *Decide* whether to reject the null hypothesis. Use a graph if necessary.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A42

EXAMPLE 5

SC Report 32

Using a Hypothesis Test for the Standard Deviation

A company claims that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes. A random sample of 25 incoming telephone calls has a standard deviation of 1.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the company's claim? Assume the population is normally distributed.

Solution

The claim is "the standard deviation is less than 1.4 minutes." So, the null and alternative hypotheses are

H_0: $\sigma \geq 1.4$ minutes and H_a: $\sigma < 1.4$ minutes. (Claim)

The test is a left-tailed test, the level of significance is $\alpha = 0.10$, and the degrees of freedom are

$$\text{d.f.} = 25 - 1 = 24.$$

So, the critical value is

$$\chi_0^2 = 15.659.$$

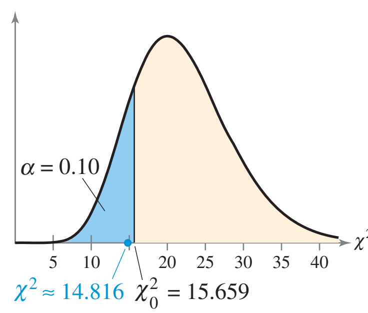
The rejection region is $\chi^2 < 15.659$. The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{Use the chi-square test.}$$

$$= \frac{(25-1)(1.1)^2}{1.4^2} \quad \text{Assume } \sigma = 1.4.$$

$$\approx 14.816.$$

The graph shows the location of the rejection region and the standardized test statistic χ^2. Because χ^2 is in the rejection region, you should reject the null hypothesis.



Interpretation There is enough evidence at the 10% level of significance to support the claim that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes.

STUDY TIP

Although you are testing a standard deviation in Example 5, the χ^2-statistic requires variances. Don't forget to square the given standard deviations to calculate these variances.

Try It Yourself 5

A police chief claims that the standard deviation of the lengths of response times is less than 3.7 minutes. A random sample of 9 response times has a standard deviation of 3.0 minutes. At $\alpha = 0.05$, is there enough evidence to support the police chief's claim? Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α and the *degrees of freedom*.
c. Find the *critical value* and identify the *rejection region*.
d. Find the *standardized test statistic* χ^2.
e. *Decide* whether to reject the null hypothesis. Use a graph if necessary.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A42

EXAMPLE 6

Using a Hypothesis Test for the Population Variance

A sporting goods manufacturer claims that the variance of the strengths of a certain fishing line is 15.9. A random sample of 15 fishing line spools has a variance of 21.8. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim? Assume the population is normally distributed.

Solution

The claim is "the variance is 15.9." So, the null and alternative hypotheses are

H_0: $\sigma^2 = 15.9$ (Claim)

and

H_a: $\sigma^2 \neq 15.9$.

The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are

$$\begin{aligned} \text{d.f.} &= 15 - 1 \\ &= 14. \end{aligned}$$

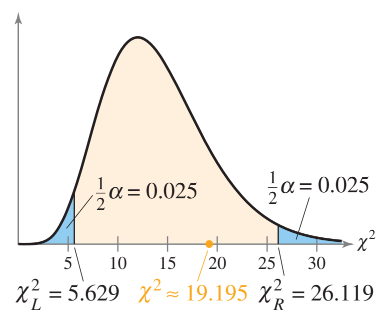
So, the critical values are $\chi^2_L = 5.629$ and $\chi^2_R = 26.119$.
The rejection regions are $\chi^2 < 5.629$ and $\chi^2 > 26.119$. The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \text{Use the chi-square test.}$$

$$= \frac{(15-1)(21.8)}{15.9} \qquad \text{Assume } \sigma^2 = 15.9.$$

$$\approx 19.195.$$

The graph shows the location of the rejection regions and the standardized test statistic χ^2. Because χ^2 is not in the rejection regions, you should fail to reject the null hypothesis.



Interpretation There is not enough evidence at the 5% level of significance to reject the claim that the variance of the strengths of the fishing line is 15.9.

Try It Yourself 6

A company that offers dieting products and weight loss services claims that the variance of the weight losses of their users is 25.5. A random sample of 13 users has a variance of 10.8. At $\alpha = 0.10$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α and the *degrees of freedom*.
c. Find the *critical values* and identify the *rejection regions*.
d. Find the *standardized test statistic* χ^2.
e. *Decide* whether to reject the null hypothesis. Use a graph if necessary.
f. *Interpret* the decision in the context of the original claim.

Answer: Page A42

7.5 EXERCISES

BUILDING BASIC SKILLS AND VOCABULARY

1. Explain how to find critical values in a χ^2- sampling distribution.

2. Can a critical value for the χ^2-test be negative? Explain.

3. When testing a claim about a population mean or a population standard deviation, a requirement is that the sample is from a population that is normally distributed. How is this requirement different between the two tests?

4. Explain how to test a population variance or a population standard deviation.

In Exercises 5–10, find the critical value(s) for the indicated test for a population variance, sample size n, and level of significance α.

5. Right-tailed test, $n = 27, \alpha = 0.05$

6. Right-tailed test, $n = 10, \alpha = 0.10$

7. Left-tailed test, $n = 7, \alpha = 0.01$

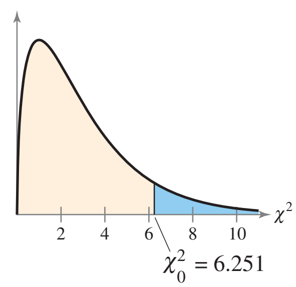
8. Left-tailed test, $n = 24, \alpha = 0.05$

9. Two-tailed test, $n = 81, \alpha = 0.10$

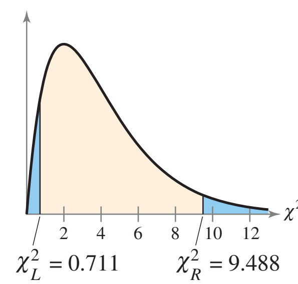
10. Two-tailed test, $n = 61, \alpha = 0.01$

Graphical Analysis *In Exercises 11–14, state whether the standardized test statistic χ^2 allows you to reject the null hypothesis.*

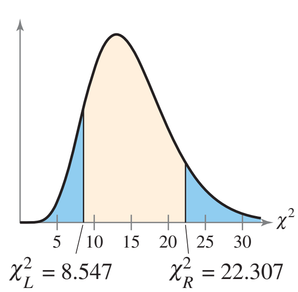
11. (a) $\chi^2 = 2.091$
(b) $\chi^2 = 0$
(c) $\chi^2 = 1.086$
(d) $\chi^2 = 6.3471$



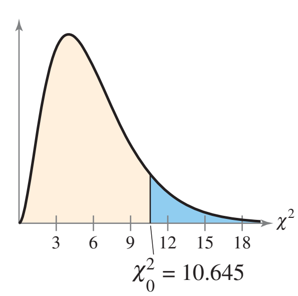
12. (a) $\chi^2 = 0.771$
(b) $\chi^2 = 9.486$
(c) $\chi^2 = 0.701$
(d) $\chi^2 = 9.508$



13. (a) $\chi^2 = 22.302$
(b) $\chi^2 = 23.309$
(c) $\chi^2 = 8.457$
(d) $\chi^2 = 8.577$



14. (a) $\chi^2 = 10.065$
(b) $\chi^2 = 10.075$
(c) $\chi^2 = 10.585$
(d) $\chi^2 = 10.745$



In Exercises 15–18, use a χ^2-test to test the claim about the population variance σ^2 or standard deviation σ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed.

15. Claim: $\sigma^2 = 0.52$; $\alpha = 0.05$. Sample statistics: $s^2 = 0.508$, $n = 18$

16. Claim: $\sigma^2 \geq 8.5$; $\alpha = 0.05$. Sample statistics: $s^2 = 7.45$, $n = 23$

17. Claim: $\sigma = 24.9$; $\alpha = 0.10$. Sample statistics: $s = 29.1$, $n = 51$

18. Claim: $\sigma < 40$; $\alpha = 0.01$. Sample statistics: $s = 40.8$, $n = 12$

USING AND INTERPRETING CONCEPTS

Testing Claims *In Exercises 19–28, (a) write the claim mathematically and identify H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic χ^2, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. For each claim, assume the population is normally distributed.*

19. Carbohydrates A snack food manufacturer estimates that the variance of the number of grams of carbohydrates in servings of its tortilla chips is 1.25. A dietician is asked to test this claim and finds that a random sample of 22 servings has a variance of 1.35. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

20. Hybrid Vehicle Gas Mileage An auto manufacturer believes that the variance of the gas mileages of its hybrid vehicles is 1.0. You work for an energy conservation agency and want to test this claim. You find that a random sample of the gas mileages of 25 of the manufacturer's hybrid vehicles has a variance of 1.65. At $\alpha = 0.05$, do you have enough evidence to reject the manufacturer's claim? *(Adapted from Green Hybrid)*

21. Science Assessment Tests On a science assessment test, the scores of a random sample of 22 eighth grade students have a standard deviation of 33.4 points. This result prompts a test administrator to claim that the standard deviation for eighth graders on the examination is less than 36 points. At $\alpha = 0.10$, is there enough evidence to support the administrator's claim? *(Adapted from National Center for Educational Statistics)*

22. U.S. History Assessment Tests A state school administrator says that the standard deviation of test scores for eighth grade students who took a U.S. history assessment test is less than 30 points. You work for the administrator and are asked to test this claim. You randomly select 18 tests and find that the tests have a standard deviation of 33.6 points. At $\alpha = 0.01$, is there enough evidence to support the administrator's claim? *(Adapted from National Center for Educational Statistics)*

23. Tornadoes A weather service claims that the standard deviation of the number of fatalities per year from tornadoes is no more than 25. A random sample of the number of deaths for 28 years has a standard deviation of 31 fatalities. At $\alpha = 0.10$, is there enough evidence to reject the weather service's claim? *(Source: NOAA Weather Partners)*

24. Lengths of Stay A doctor says the standard deviation of the lengths of stay for patients involved in a crash in which the vehicle struck a tree is 6.14 days. A random sample of 20 lengths of stay for patients involved in this type of crash has a standard deviation of 6.5 days. At $\alpha = 0.05$, can you reject the doctor's claim? *(Adapted from National Highway Traffic Safety Administration)*

25. Total Charges An insurance agent says the standard deviation of the total hospital charges for patients involved in a crash in which the vehicle struck a construction barricade is less than \$3500. A random sample of 28 total hospital charges for patients involved in this type of crash has a standard deviation of \$4100. At $\alpha = 0.10$, can you support the agent's claim? *(Adapted from National Highway Traffic Safety Administration)*

26. Hotel Room Rates A travel agency estimates that the standard deviation of the room rates of hotels in a certain city is no more than \$30. You work for a consumer advocacy group and are asked to test this claim. You find that a random sample of 21 hotels has a standard deviation of \$35.25. At $\alpha = 0.01$, do you have enough evidence to reject the agency's claim?

27. Salaries The annual salaries (in dollars) of 18 randomly chosen environmental engineers are listed. At $\alpha = 0.05$, can you conclude that the standard deviation of the annual salaries is greater than \$6100? *(Adapted from Salary.com)*

63,125	59,749	52,369	55,979	61,550	54,644	50,420
47,291	51,357	56,901	53,499	49,998	69,712	64,575
45,850	46,297	63,770	71,589			

28. Salaries A staffing organization states that the standard deviation of the annual salaries of commodity buyers is at least \$10,600. The annual salaries (in dollars) of 20 randomly chosen commodity buyers are listed. At $\alpha = 0.10$, can you reject the organization's claim? *(Adapted from Salary.com)*

79,319	68,825	65,129	75,899	85,070	76,270	68,750
70,982	69,237	63,470	79,025	55,880	80,985	75,264
66,918	65,459	70,598	86,579	71,225	57,311	

SC *In Exercises 29–32, use StatCrunch to help you test the claim about the population variance σ^2 or standard deviation σ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed.*

29. Claim: $\sigma^2 \geq 9$; $\alpha = 0.01$. Sample statistics: $s^2 = 2.03$, $n = 10$

30. Claim: $\sigma^2 = 14.85$; $\alpha = 0.05$. Sample statistics: $s^2 = 28.75$, $n = 17$

31. Claim: $\sigma > 4.5$; $\alpha = 0.05$. Sample statistics: $s = 5.8$, $n = 15$

32. Claim: $\sigma \neq 418$; $\alpha = 0.10$. Sample statistics: $s = 305$, $n = 24$

EXTENDING CONCEPTS

P-Values *You can calculate the P-value for a χ^2-test using technology. After calculating the χ^2-test value, you can use the cumulative density function (CDF) to calculate the area under the curve. From Example 4 on page 407, $\chi^2 = 43.2$. Using a TI-83/84 Plus (choose 7 from the DISTR menu), enter 0 for the lower bound, 43.2 for the upper bound, and 40 for the degrees of freedom, as shown at the left.*

TI-83/84 PLUS

χ^2 cdf (0, 43.2, 40)
.6637768667

The P-value is approximately $1 - 0.6638 = 0.3362$. Because $P > \alpha = 0.05$, the conclusion is to fail to reject H_0.

In Exercises 33–36, use the P-value method to perform the hypothesis test for the indicated exercise.

33. Exercise 25

34. Exercise 26

35. Exercise 27

36. Exercise 28

USES AND ABUSES

Uses

Hypothesis Testing Hypothesis testing is important in many different fields because it gives a scientific procedure for assessing the validity of a claim about a population. Some of the concepts in hypothesis testing are intuitive, but some are not. For instance, the *American Journal of Clinical Nutrition* suggests that eating dark chocolate can help prevent heart disease. A random sample of healthy volunteers were assigned to eat 3.5 ounces of dark chocolate each day for 15 days. After 15 days, the mean systolic blood pressure of the volunteers was 6.4 millimeters of mercury lower. A hypothesis test could show if this drop in systolic blood pressure is significant or simply due to sampling error.

Careful inferences must be made concerning the results. In another part of the study, it was found that white chocolate did not result in similar benefits. So, the inference of health benefits cannot be extended to all types of chocolate. You also would not infer that you should eat large quantities of chocolate because the benefits must be weighed against known risks, such as weight gain, acne, and acid reflux.

Abuses

Not Using a Random Sample The entire theory of hypothesis testing is based on the fact that the sample is randomly selected. If the sample is not random, then you cannot use it to infer anything about a population parameter.

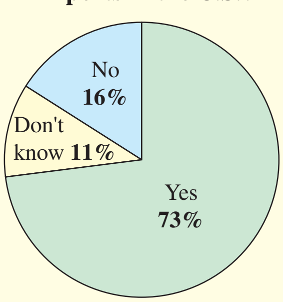
Attempting to Prove the Null Hypothesis If the P-value for a hypothesis test is greater than the level of significance, you have not proven the null hypothesis is true—only that there is not enough evidence to reject it. For instance, with a P-value higher than the level of significance, a researcher could not prove that there is no benefit to eating dark chocolate—only that there is not enough evidence to support the claim that there is a benefit.

Making Type I or Type II Errors Remember that a type I error is rejecting a null hypothesis that is true and a type II error is failing to reject a null hypothesis that is false. You can decrease the probability of a type I error by lowering the level of significance. Generally, if you decrease the probability of making a type I error, you increase the probability of making a type II error. You can decrease the chance of making both types of errors by increasing the sample size.

EXERCISES

In Exercises 1–4, assume that you work in a transportation department. You are asked to write a report about the claim that 73% of U.S. adults who fly at least once a year favor full-body scanners at airports. (Adapted from Rasmussen Reports)

Do You Favor the Use of Full-Body Scanners at Airports in the U.S.?



1. ***Not Using a Random Sample*** How could you choose a random sample to test this hypothesis?

2. ***Attempting to Prove the Null Hypothesis*** What is the null hypothesis in this situation? Describe how your report could be incorrect by trying to prove the null hypothesis.

3. ***Making a Type I Error*** Describe how your report could make a type I error.

4. ***Making a Type II Error*** Describe how your report could make a type II error.

7 A SUMMARY OF HYPOTHESIS TESTING

With hypothesis testing, perhaps more than any other area of statistics, it can be difficult to see the forest for all the trees. To help you see the forest—the overall picture—a summary of what you studied in this chapter is provided.

Writing the Hypotheses

- You are given a claim about a population parameter μ, p, σ^2, or σ.
- Rewrite the claim and its complement using $\underbrace{\le, \ge, =}_{H_0}$ and $\underbrace{>, <, \neq}_{H_a}$.
- Identify the claim. Is it H_0 or H_a?

Specifying a Level of Significance

- Specify α, the maximum acceptable probability of rejecting a valid H_0 (a type I error).

Specifying the Sample Size

- Specify your sample size n.

INSIGHT

Large sample sizes will usually increase the cost and effort of testing a hypothesis, but they also tend to make your decision more reliable.

Choosing the Test ■ Any population ■ Normally distributed population

- Mean: H_0 describes a hypothesized population mean μ.
  - Use a ***z*-test** for *any* population if $n \ge 30$.
  - Use a ***z*-test** if the population is normal and σ is known for any n.
  - Use a ***t*-test** if the population is normal and $n < 30$, but σ is unknown.
- **Proportion:** H_0 describes a hypothesized population proportion p.
  - Use a ***z*-test** for any population if $np \ge 5$ and $nq \ge 5$.
- **Variance or Standard Deviation:** H_0 describes a hypothesized population variance σ^2 or standard deviation σ.
  - Use a **χ^2-test** if the population is normal.

Sketching the Sampling Distribution

- Use H_a to decide if the test is left-tailed, right-tailed, or two-tailed.

Finding the Standardized Test Statistic

- Take a random sample of size n from the population.
- Compute the test statistic $\bar{x}$, $\hat{p}$, or s^2.
- Find the standardized test statistic z, t, or χ^2.

Making a Decision

Option 1. Decision based on rejection region

- Use α to find the critical value(s) z_0, t_0, or χ_0^2 and rejection region(s).
- **Decision Rule:**

  Reject H_0 if the standardized test statistic is in the rejection region.
  Fail to reject H_0 if the standardized test statistic is not in the rejection region.

Option 2. Decision based on P-value

- Use the standardized test statistic or a technology tool to find the P-value.
- **Decision Rule:**

  Reject H_0 if $P \le \alpha$.
  Fail to reject H_0 if $P > \alpha$.

z-Test for a Hypothesized Mean μ (Section 7.2)

Test statistic: $\bar{x}$

Critical value: z_0 (Use Table 4.)

If $n \geq 30$, s can be used in place of σ.

Sampling distribution of sample means is a normal distribution.

Standardized test statistic: z

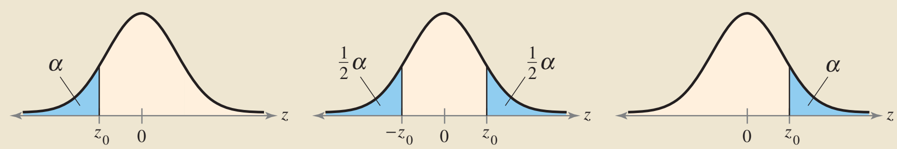
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Sample mean

Hypothesized mean

Population standard deviation

Sample size



Left-Tailed Two-Tailed Right-Tailed

z-Test for a Hypothesized Proportion p (Section 7.4)

Test statistic: $\hat{p}$

Critical value: z_0 (Use Table 4.)

Sampling distribution of sample proportions is a normal distribution.

Standardized test statistic: z

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Sample proportion

Hypothesized proportion

$q = 1 - p$

Sample size

STUDY TIP

If your standardized test statistic is z or t, remember that these values measure standard deviations from the mean. Values that are outside of ± 3 indicate that H_0 is very unlikely. Values that are outside of ± 5 indicate that H_0 is almost impossible.

t-Test for a Hypothesized Mean μ (Section 7.3)

Test statistic: $\bar{x}$

Critical value: t_0 (Use Table 5.)

Sampling distribution of sample means is approximated by a t-distribution with d.f. $= n - 1$.

Standardized test statistic: t

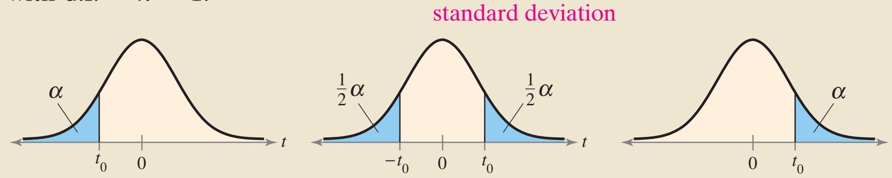
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Sample mean

Hypothesized mean

Sample standard deviation

Sample size



Left-Tailed Two-Tailed Right-Tailed

χ^2-Test for a Hypothesized Variance σ^2 or Standard Deviation σ (Section 7.5)

Test statistic: s^2

Critical value: χ^2_0 (Use Table 6.)

Sampling distribution is approximated by a chi-square distribution with d.f. $= n - 1$.

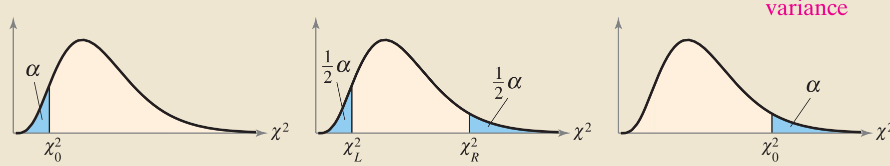
Standardized test statistic: χ^2

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Sample size

Sample variance

Hypothesized variance



Left-Tailed Two-Tailed Right-Tailed

7 CHAPTER SUMMARY

What did you learn?	EXAMPLE(S)	REVIEW EXERCISES
Section 7.1		
■ How to state a null hypothesis and an alternative hypothesis	*1*	*1–6*
■ How to identify type I and type II errors	*2*	*7–10*
■ How to know whether to use a one-tailed or a two-tailed statistical test	*3*	*7–10*
■ How to interpret a decision based on the results of a statistical test	*4*	*7–10*
Section 7.2		
■ How to find P-values and use them to test a mean μ	*1–3*	*11, 12*
■ How to use P-values for a z-test	*4–6*	*13, 14, 23–28*
■ How to find critical values and rejection regions in a normal distribution	*7, 8*	*15–18*
■ How to use rejection regions for a z-test	*9, 10*	*19–28*
Section 7.3		
■ How to find critical values in a t-distribution	*1–3*	*29–32*
■ How to use the t-test to test a mean μ	*4, 5*	*33–40*
■ How to use technology to find P-values and use them with a t-test to test a mean μ	*6*	*41, 42*
Section 7.4		
■ How to use the z-test to test a population proportion p	*1, 2*	*43–52*
Section 7.5		
■ How to find critical values for a χ^2-test	*1–3*	*53–56*
■ How to use the χ^2-test to test a variance or a standard deviation	*4–6*	*57–63*

7 REVIEW EXERCISES

SECTION 7.1

In Exercises 1–6, use the given statement to represent a claim. Write its complement and state which is H_0 and which is H_a.

1. $\mu \leq 375$

2. $\mu = 82$

3. $p < 0.205$

4. $\mu \neq 150{,}020$

5. $\sigma > 1.9$

6. $p \geq 0.64$

In Exercises 7–10, do the following.

(a) State the null and alternative hypotheses, and identify which represents the claim.

(b) Determine when a type I or type II error occurs for a hypothesis test of the claim.

(c) Determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed. Explain your reasoning.

(d) Explain how you should interpret a decision that rejects the null hypothesis.

(e) Explain how you should interpret a decision that fails to reject the null hypothesis.

7. A news outlet reports that the proportion of Americans who support plans to order deep cuts in executive compensation at companies that have received federal bailout funds is 71%. *(Source: ABC News)*

8. An agricultural cooperative guarantees that the mean shelf life of a certain type of dried fruit is at least 400 days.

9. A soup maker says that the standard deviation of the sodium content in one serving of a certain soup is no more than 50 milligrams. *(Adapted from Consumer Reports)*

10. An energy bar maker claims that the mean number of grams of carbohydrates in one bar is less than 25.

SECTION 7.2

In Exercises 11 and 12, find the P-value for the indicated hypothesis test with the given standardized test statistic z. Decide whether to reject H_0 for the given level of significance α.

11. Left-tailed test, $z = -0.94$, $\alpha = 0.05$

12. Two-tailed test, $z = 2.57$, $\alpha = 0.10$

In Exercises 13 and 14, use a P-value to test the claim about the population mean μ using the given sample statistics. State your decision for $\alpha = 0.10$, $\alpha = 0.05$, and $\alpha = 0.01$ levels of significance. If convenient, use technology.

13. Claim: $\mu \leq 0.05$; Sample statistics: $\bar{x} = 0.057$, $s = 0.018$, $n = 32$

14. Claim: $\mu \neq 230$; Sample statistics: $\bar{x} = 216.5$, $s = 17.3$, $n = 48$

In Exercises 15–18, find the critical value(s) for the indicated z-test and level of significance α. Include a graph with your answer.

15. Left-tailed test, $\alpha = 0.02$

16. Two-tailed test, $\alpha = 0.005$

17. Right-tailed test, $\alpha = 0.025$

18. Two-tailed test, $\alpha = 0.08$

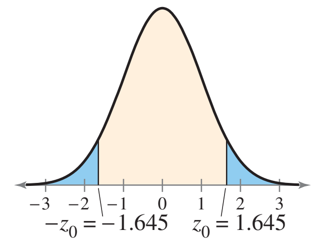
In Exercises 19–22, state whether each standardized test statistic z allows you to reject the null hypothesis. Explain your reasoning.

19. $z = 1.631$

20. $z = 1.723$

21. $z = -1.464$

22. $z = -1.655$



In Exercises 23–26, use a z-test to test the claim about the population mean μ at the given level of significance α using the given sample statistics. If convenient, use technology.

23. Claim: $\mu \leq 45$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 47.2$, $s = 6.7$, $n = 42$

24. Claim: $\mu \neq 8.45$; $\alpha = 0.03$. Sample statistics: $\bar{x} = 7.88$, $s = 1.75$, $n = 60$

25. Claim: $\mu < 5.500$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 5.497$, $s = 0.011$, $n = 36$

26. Claim: $\mu = 7450$; $\alpha = 0.10$. Sample statistics: $\bar{x} = 7495$, $s = 243$, $n = 57$

In Exercises 27 and 28, test the claim about the population mean μ using rejection region(s) or a P-value. Interpret your decision in the context of the original claim. If convenient, use technology.

27. The U.S. Department of Agriculture claims that the mean cost of raising a child from birth to age 2 by husband-wife families in rural areas is \$10,380. A random sample of 800 children (age 2) has a mean cost of \$10,240 with a standard deviation of \$1561. At $\alpha = 0.01$, is there enough evidence to reject the claim? *(Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion)*

28. A tourist agency in Hawaii claims the mean daily cost of meals and lodging for a family of 4 traveling in Hawaii is at most \$650. You work for a consumer protection advocate and want to test this claim. In a random sample of 45 families of 4 traveling in Hawaii, the mean daily cost of meals and lodging is \$657 with a standard deviation of \$40. At $\alpha = 0.05$, do you have enough evidence to reject the tourist agency's claim? *(Adapted from American Automobile Association)*

SECTION 7.3

In Exercises 29–32, find the critical value(s) for the indicated t-test, level of significance α, and sample size n.

29. Two-tailed test, $\alpha = 0.05$, $n = 20$

30. Right-tailed test, $\alpha = 0.01$, $n = 8$

31. Left-tailed test, $\alpha = 0.005$, $n = 15$

32. Two-tailed test, $\alpha = 0.02$, $n = 12$

In Exercises 33–38, use a t-test to test the claim about the population mean μ at the given level of significance α using the given sample statistics. For each claim, assume the population is normally distributed. If convenient, use technology.

33. Claim: $\mu \neq 95$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 94.1$, $s = 1.53$, $n = 12$

34. Claim: $\mu > 12{,}700$; $\alpha = 0.005$. Sample statistics: $\bar{x} = 12{,}855$, $s = 248$, $n = 21$

35. Claim: $\mu \geq 0$; $\alpha = 0.10$. Sample statistics: $\bar{x} = -0.45$, $s = 1.38$, $n = 16$

36. Claim: $\mu = 4.20$; $\alpha = 0.02$. Sample statistics: $\bar{x} = 4.61$, $s = 0.33$, $n = 9$

37. Claim: $\mu \leq 48$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 52$, $s = 2.5$, $n = 7$

38. Claim: $\mu < 850$; $\alpha = 0.025$. Sample statistics: $\bar{x} = 875$, $s = 25$, $n = 14$

In Exercises 39 and 40, use a t-test to test the claim. Interpret your decision in the context of the original claim. For each claim, assume the population is normally distributed. If convenient, use technology.

39. A fitness magazine advertises that the mean monthly cost of joining a health club is \$25. You work for a consumer advocacy group and are asked to test this claim. You find that a random sample of 18 clubs has a mean monthly cost of \$26.25 and a standard deviation of \$3.23. At $\alpha = 0.10$, do you have enough evidence to reject the advertisement's claim?

40. A fitness magazine claims that the mean cost of a yoga session is no more than \$14. You work for a consumer advocacy group and are asked to test this claim. You find that a random sample of 29 yoga sessions has a mean cost of \$15.59 and a standard deviation of \$2.60. At $\alpha = 0.025$, do you have enough evidence to reject the magazine's claim?

In Exercises 41 and 42, use a t-statistic and its P-value to test the claim about the population mean μ using the given data. Interpret your decision in the context of the original claim. For each claim, assume the population is normally distributed. If convenient, use technology.

41. An education publication claims that the mean expenditure per student in public elementary and secondary schools is at least \$10,200. You want to test this claim. You randomly select 16 school districts and find the average expenditure per student. The results are listed below. At $\alpha = 0.01$, can you reject the publication's claim? *(Adapted from National Center for Education Statistics)*

9,242	10,857	10,377	8,935	9,545	9,974
9,847	10,641	9,364	10,157	9,784	9,962
10,065	9,851	9,763	9,969		

42. A restaurant association says the typical household in the United States spends a mean amount of \$2698 per year on food away from home. You are a consumer reporter for a national publication and want to test this claim. A random sample of 28 U.S. households has a mean amount spent on food away from home of \$2764 and a standard deviation of \$322. At $\alpha = 0.05$, do you have enough evidence to reject the association's claim? *(Adapted from U.S. Bureau of Labor Statistics)*

SECTION 7.4

In Exercises 43–50, decide whether the normal sampling distribution can be used to approximate the binomial distribution. If it can, use the z-test to test the claim about the population proportion p at the given level of significance α using the given sample statistics. If convenient, use technology.

43. Claim: $p = 0.15$; $\alpha = 0.05$. Sample statistics: $\hat{p} = 0.09$, $n = 40$

44. Claim: $p < 0.70$; $\alpha = 0.01$. Sample statistics: $\hat{p} = 0.50$, $n = 68$

45. Claim: $p < 0.09$; $\alpha = 0.08$. Sample statistics: $\hat{p} = 0.07$, $n = 75$

46. Claim: $p = 0.65$; $\alpha = 0.03$. Sample statistics: $\hat{p} = 0.76$, $n = 116$

47. Claim: $p \geq 0.04$; $\alpha = 0.10$. Sample statistics: $\hat{p} = 0.03$, $n = 30$

48. Claim: $p \neq 0.34$; $\alpha = 0.01$. Sample statistics: $\hat{p} = 0.29$, $n = 60$

49. Claim: $p \neq 0.24$; $\alpha = 0.02$. Sample statistics: $\hat{p} = 0.32$, $n = 50$

50. Claim: $p \leq 0.80$; $\alpha = 0.10$. Sample statistics: $\hat{p} = 0.85$, $n = 43$

In Exercises 51 and 52, test the claim about the population proportion p. Interpret your decision in the context of the original claim. If convenient, use technology.

51. A polling agency reports that over 16% of U.S. adults are without health care coverage. In a random survey of 1420 U.S. adults, 256 said they did not have health care coverage. At $\alpha = 0.02$, is there enough evidence to support the agency's claim? *(Source: The Gallup Poll)*

52. The Western blot assay is a blood test for the presence of HIV. It has been found that this test sometimes gives false positive results for HIV. A medical researcher claims that the rate of false positives is 2%. A recent study of 300 randomly selected U.S. blood donors who do not have HIV found that 3 received a false positive test result. At $\alpha = 0.05$, is there enough evidence to reject the researcher's claim? *(Adapted from Centers for Disease Control and Prevention)*

SECTION 7.5

In Exercises 53–56, find the critical value(s) for the indicated χ^2-test for a population variance, sample size n, and level of significance α.

53. Right-tailed test, $n = 20$, $\alpha = 0.05$

54. Two-tailed test, $n = 14$, $\alpha = 0.01$

55. Right-tailed test, $n = 51$, $\alpha = 0.10$

56. Left-tailed test, $n = 6$, $\alpha = 0.05$

In Exercises 57–60, use a χ^2-test to test the claim about the population variance σ^2 or standard deviation σ at the given level of significance α and using the given sample statistics. For each claim, assume the population is normally distributed.

57. Claim: $\sigma^2 > 2$; $\alpha = 0.10$. Sample statistics: $s^2 = 2.95$, $n = 18$

58. Claim: $\sigma^2 \leq 60$; $\alpha = 0.025$. Sample statistics: $s^2 = 72.7$, $n = 15$

59. Claim: $\sigma = 1.25$; $\alpha = 0.05$. Sample statistics: $s = 1.03$, $n = 6$

60. Claim: $\sigma \neq 0.035$; $\alpha = 0.01$. Sample statistics: $s = 0.026$, $n = 16$

In Exercises 61 and 62, test the claim about the population variance or standard deviation. Interpret your decision in the context of the original claim. For each claim, assume the population is normally distributed.

61. A bolt manufacturer makes a type of bolt to be used in airtight containers. The manufacturer needs to be sure that all of its bolts are very similar in width, so it sets an upper tolerance limit for the variance of bolt width at 0.01. A random sample of the widths of 28 bolts has a variance of 0.064. At $\alpha = 0.005$, is there enough evidence to reject the manufacturer's claim?

62. A restaurant claims that the standard deviation of the lengths of serving times is 3 minutes. A random sample of 27 serving times has a standard deviation of 3.9 minutes. At $\alpha = 0.01$, is there enough evidence to reject the restaurant's claim?

63. In Exercise 62, is there enough evidence to reject the restaurant's claim at the $\alpha = 0.05$ level? Explain.

7 CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book. If convenient, use technology.

For this quiz, do the following.

(a) Write the claim mathematically. Identify H_0 and H_a.

(b) Determine whether the hypothesis test is one-tailed or two-tailed and whether to use a z-test, a t-test, or a χ^2-test. Explain your reasoning.

(c) If necessary, find the critical value(s) and identify the rejection region(s).

(d) Find the appropriate test statistic. If necessary, find the P-value.

(e) Decide whether to reject or fail to reject the null hypothesis.

(f) Interpret the decision in the context of the original claim.

1. A research service estimates that the mean annual consumption of vegetables and melons by people in the United States is at least 170 pounds per person. A random sample of 360 people in the United States has a mean consumption of vegetables and melons of 168.5 pounds per year and a standard deviation of 11 pounds. At $\alpha = 0.03$, is there enough evidence to reject the service's claim that the mean consumption of vegetables and melons by people in the United States is at least 170 pounds per person? *(Adapted from U.S. Department of Agriculture)*

2. A hat company states that the mean hat size for a male is at least 7.25. A random sample of 12 hat sizes has a mean of 7.15 and a standard deviation of 0.27. At $\alpha = 0.05$, can you reject the company's claim that the mean hat size for a male is at least 7.25? Assume the population is normally distributed.

3. A maker of microwave ovens advertises that no more than 10% of its microwaves need repair during the first 5 years of use. In a random sample of 57 microwaves that are 5 years old, 13% needed repairs. At $\alpha = 0.04$, can you reject the maker's claim that no more than 10% of its microwaves need repair during the first five years of use? *(Adapted from Consumer Reports)*

4. A state school administrator says that the standard deviation of SAT critical reading test scores is 112. A random sample of 19 SAT critical reading test scores has a standard deviation of 143. At $\alpha = 0.10$, test the administrator's claim. What can you conclude? Assume the population is normally distributed. *(Adapted from The College Board)*

5. A government agency reports that the mean amount of earnings for full-time workers ages 25 to 34 with a master's degree is \$62,569. In a random sample of 15 full-time workers ages 25 to 34 with a master's degree, the mean amount of earnings is \$59,231 and the standard deviation is \$5945. Is there enough evidence to reject the agency's claim? Use a P-value and $\alpha = 0.05$. Assume the population is normally distributed. *(Adapted from U.S. Census Bureau)*

6. A tourist agency in Kansas claims the mean daily cost of meals and lodging for a family of 4 traveling in the state is \$201. You work for a consumer protection advocate and want to test this claim. In a random sample of 35 families of 4 traveling in Kansas, the mean daily cost of meals and lodging is \$216 and the standard deviation is \$30. Do you have enough evidence to reject the agency's claim? Use a P-value and $\alpha = 0.05$. *(Adapted from American Automobile Association)*

PUTTING IT ALL TOGETHER

Real Statistics — Real Decisions

In the 1970s and 1980s, PepsiCo, maker of Pepsi®, began airing television commercials in which it claimed more cola drinkers preferred Pepsi® over Coca-Cola® in a blind taste test. The Coca-Cola Company, maker of Coca-Cola®, was the market leader in soda sales. After the television ads began airing, Pepsi® sales increased and began rivaling Coca-Cola® sales.

Assume the claim is that more than 50% of cola drinkers preferred Pepsi® over Coca-Cola®. You work for an independent market research firm and are asked to test this claim.

EXERCISES

1. ***How Would You Do It?***
   (a) When PepsiCo performed this challenge, PepsiCo representatives went to shopping malls to obtain their sample. Do you think this type of sampling is representative of the population? Explain.
   (b) What sampling technique would you use to select the sample for your study?
   (c) Identify possible flaws or biases in your study.

2. ***Testing a Proportion***
   In your study, 280 out of 560 cola drinkers prefer Pepsi® over Coca-Cola®. Using these results, test the claim that more than 50% of cola drinkers prefer Pepsi® over Coca-Cola®. Use $\alpha = 0.05$. Interpret your decision in the context of the original claim. Does the decision support PepsiCo's claim?

3. ***Labeling Influence***
   The Baylor College of Medicine decided to replicate this taste test by monitoring brain activity while conducting the test on participants. They also wanted to see if brand labeling would affect the results. When participants were shown which cola they were sampling, Coca-Cola® was preferred by 75% of the participants. What conclusions can you draw from this study?

4. ***Your Conclusions***
   (a) Why do you think PepsiCo used a blind taste test?
   (b) Do you think brand image or taste has more influence on consumer preferences for cola?
   (c) What other factors may influence consumer preferences besides taste and branding?

TECHNOLOGY

MINITAB EXCEL TI-83/84 PLUS

THE CASE OF THE VANISHING WOMEN

53% → 29% → 9% → 0%

From 1966 to 1968, Dr. Benjamin Spock and others were tried for conspiracy to violate the Selective Service Act by encouraging resistance to the Vietnam War. By a series of three selections, no women ended up being on the jury. In 1969, Hans Zeisel wrote an article in *The University of Chicago Law Review* using statistics and hypothesis testing to argue that the jury selection was biased against Dr. Spock. Dr. Spock was a well-known pediatrician and author of books about raising children. Millions of mothers had read his books and followed his advice. Zeisel argued that, by keeping women off the jury, the court prejudiced the verdict.

The jury selection process for Dr. Spock's trial is shown at the right.

Stage 1. The clerk of the Federal District Court selected 350 people "at random" from the Boston City Directory. The directory contained several hundred names, 53% of whom were women. However, only 102 of the 350 people selected were women.

Stage 2. The trial judge, Judge Ford, selected 100 people "at random" from the 350 people. This group was called a venire and it contained only nine women.

Stage 3. The court clerk assigned numbers to the members of the venire and, one by one, they were interrogated by the attorneys for the prosecution and defense until 12 members of the jury were chosen. At this stage, only one potential female juror was questioned, and she was eliminated by the prosecutor under his quota of peremptory challenges (for which he did not have to give a reason).

EXERCISES

1. The MINITAB display below shows a hypothesis test for a claim that the proportion of women in the city directory is $p = 0.53$. In the test, $n = 350$ and $\hat{p} \approx 0.2914$. Should you reject the claim? What is the level of significance? Explain.

2. In Exercise 1, you rejected the claim that $p = 0.53$. But this claim was true. What type of error is this?

3. If you reject a true claim with a level of significance that is virtually zero, what can you infer about the randomness of your sampling process?

4. Describe a hypothesis test for Judge Ford's "random" selection of the venire. Use a claim of

$$p = \frac{102}{350} \approx 0.2914.$$

(a) Write the null and alternative hypotheses.

(b) Use a technology tool to perform the test.

(c) Make a decision.

(d) Interpret the decision in the context of the original claim. Could Judge Ford's selection of 100 venire members have been random?

MINITAB

Test and CI for One Proportion

Test of p = 0.53 vs p not = 0.53

Sample	X	N	Sample p	99 % CI	Z-Value	P-Value
1	102	350	0.291429	(0.228862, 0.353995)	−8.94	0.000

Using the normal approximation.

Extended solutions are given in the *Technology Supplement*.
Technical instruction is provided for MINITAB, Excel, and the TI-83/84 Plus.

7 USING TECHNOLOGY TO PERFORM HYPOTHESIS TESTS

Here are some MINITAB and TI-83/84 Plus printouts for some of the examples in this chapter.

(See Example 5, page 375.)

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

One-Sample Z

Test of mu = 22500 vs not = 22500
The assumed standard deviation = 3015

N	Mean	SE Mean	95% CI	Z	P
30	21545	550	(20466, 22624)	−1.73	0.083

(See Example 4, page 390.)

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

One-Sample T

Test of mu = 20500 vs < 20500

N	Mean	StDev	SE Mean	95% Upper Bound	T	P
14	19850	1084	290	20363	−2.24	0.021

(See Example 2, page 400.)

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

Test and CI for One Proportion

Test of p = 0.25 vs p not = 0.25

Sample	X	N	Sample p	90% CI	Z-Value	P-Value
1	42	200	0.210000	(0.162627, 0.257373)	−1.31	0.191

Using the normal approximation.

(See Example 9, page 379.)

TI-83/84 PLUS

EDIT CALC **TESTS**

1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ZInterval...

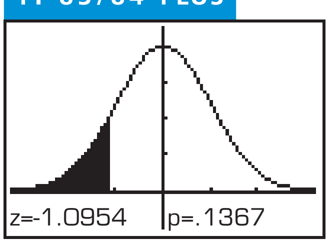
TI-83/84 PLUS

Z-Test
Inpt: Data **Stats**
μ_0: 68000
σ: 5500
$\bar{x}$: 66900
n: 30
μ: $\neq\mu_0$ **$<\mu_0$** $>\mu_0$
Calculate Draw

TI-83/84 PLUS

Z-Test
$\mu<68000$
z= −1.095445115
p= .1366608782
$\bar{x}$= 66900
n= 30

TI-83/84 PLUS



(See Example 5, page 391.)

TI-83/84 PLUS

EDIT CALC **TESTS**

1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ZInterval...

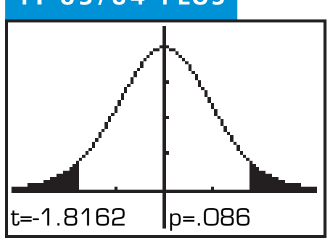
TI-83/84 PLUS

T-Test
Inpt: Data **Stats**
μ_0: 6.8
$\bar{x}$: 6.7
Sx: .24
n: 19
μ: **$\neq\mu_0$** $<\mu_0$ $>\mu_0$
Calculate Draw

TI-83/84 PLUS

T-Test
$\mu\neq 6.8$
t= −1.816207893
p= .0860316039
$\bar{x}$= 6.7
Sx= .24
n= 19

TI-83/84 PLUS



(See Example 1, page 399.)

TI-83/84 PLUS

EDIT CALC **TESTS**

1: Z–Test...
2: T–Test...
3: 2–SampZTest...
4: 2–SampTTest...
5: 1–PropZTest...
6: 2–PropZTest...
7↓ZInterval...

TI-83/84 PLUS

1-PropZTest
p_0: .5
x: 39
n: 100
prop$\neq p_0$ **$<p_0$** $>p_0$
Calculate Draw

TI-83/84 PLUS

1-PropZTest
prop< .5
z= −2.2
p= .0139033989
$\hat{p}$= .39
n= 100

TI-83/84 PLUS